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# THE DEVELOPMENT OF ROOF SOLAR PANELS

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*This paper suggests electric generating coating material in the form of tile made of recycled materials based on solar cells and concentrator of solar radiation. Two versions of solar tiles – version without concentrators with planar solar cells and gel increasing their service life, and tiles with concentrators, sliced solar cells, gel, concentrator of solar radiation and an optical deflection system in the form of a prism. Alongside with importance for construction, solar tiles can solve the problem of power generation, increase service life of solar modules and secondary use of plastic waste.*

***Keywords:*** *electricity supply, solar tiles, concentrator of solar radiation, polysiloxane compound, secondary raw materials.*

Currently, one of the most relevant trends in construction and architecture is improvement of energy performance of buildings and integration of plants using renewable energy sources into building structures. In this case a building equipped with systems for solar energy conversion can be considered a passive system for solar energy supply [1].

There is a wide range of technological solutions for rational use of solar radiation available for a building. One of such solutions is location of large windows at the south side of a façade in order to save electric energy needed for room illumination and better warming by solar radiation in winter season. For regulation of incoming solar radiation window blinds are used operating in both automatic and manual modes. To provide illumination to rooms lacking the possibility of natural lighting, fiber-optical waveguides are used. Another version of architectural solutions for electricity supply to residential building are solar modules integrated into roof covering – the so-called «solar tiles», as the main disadvantage of widely used solar modules is the necessity of installing roof covering under such a module in order to protect buildings from external factors, which increases buildings cost.

Solar tiles are used as roof covering in construction of buildings in combination with power generation from solar radiation. The use of solar tiles can solve architectural and construction problems, as well as the problem of electric energy supply to consumers, both autonomous or parallel with the power network. Moreover, in fabrication of solar tiles plastic waste is used. This one product replaces two products (conventional covering tiles and a photovoltaic module) combining both functions (protection and power generation). Solar tiles are tiles of standard form, made of recyclable materials (polyethylene bottles or film and binding materials, which reduces production cost and is beneficial for environment. Solar tiles also comprise solar cells placed in polysiloxane compound increasing their useful life period, and working in combination with an add-on concentrator (in dependence on the region wherein solar tiles are used). When installed, solar tiles are fixed to a timber bean by two self-tapping screws each, covering a part of the next tile, thus forming a lock. Planar solar cells encapsulated by organosilicon two-component polysiloxane compound prolonging solar cell useful life up to 40-50 years, are used as electric power generating components [2, 3].

When direct solar radiation component is high (southern, Siberian and mountainous regions, clean atmosphere), alongside with solar cells, solar radiation concentrators are used, which reduces the price of solar modules due to solar-grade silicon saving. If overcast days and days with low solar radiation prevail, solar tiles without concentrators with standard planar solar cells are used, providing annual electric power generation comparable to electricity generation by solar tiles with concentrators in regions where direct solar radiation component is high.

Sequential commutation of solar tiles is used in order to obtain higher voltage at the output. Each tile voltage amounts to 1-1.2 V and current to 7-8 A.

For the implementation of the idea of solar tiles, two-dimensional model of planar and concentrator tiles has been developed in AutoCad (Fig. 1) - the two and three-dimensional computer-aided design and drafting system widely used in machine-building, construction, architecture and other industries [4].

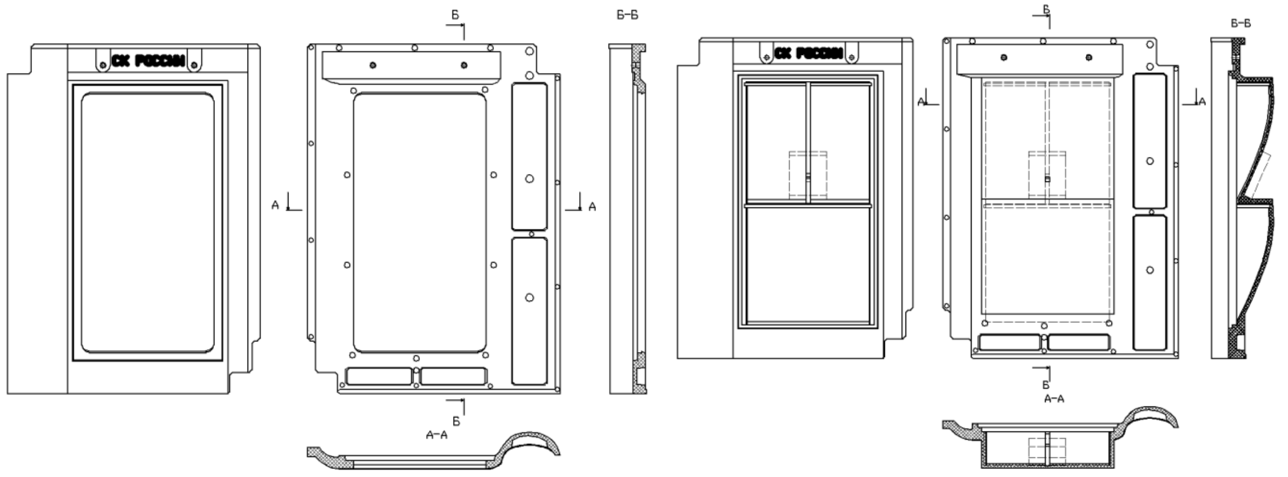


**Fig. 1. Drawings of solar tiles – planar (to the left) and with a concentrator cavity (to the right)**

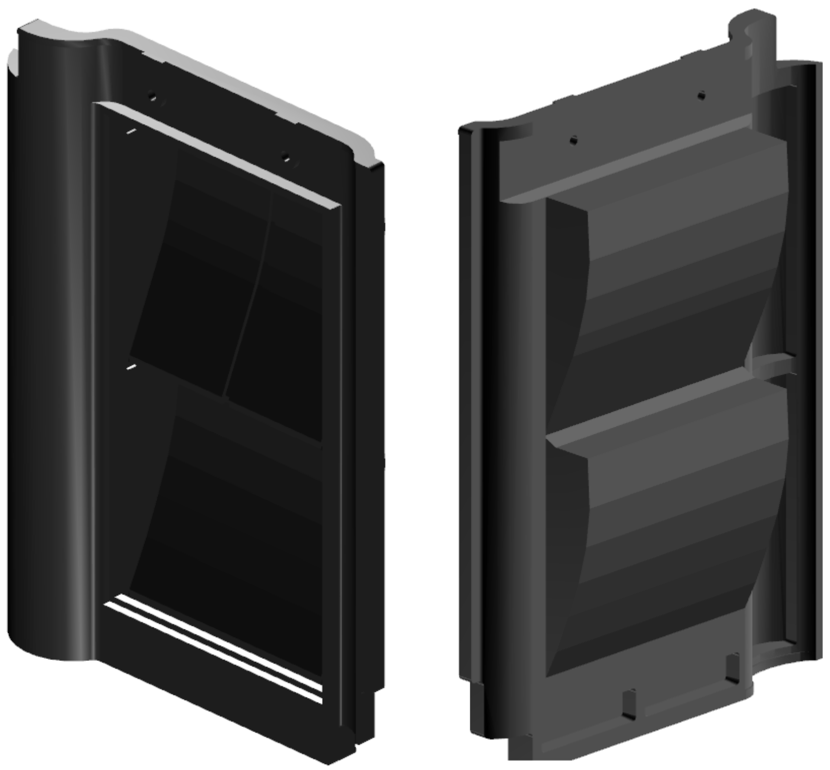

**Fig. 2. Three-dimensional model of solar tiles with a concentrator cavity**

In designing the model of solar tiles, the versatility of the model for fabrication of various types of solar tiles (with various sizes of solar cells, with a concentrator cavity, etc.) was considered. For fabricating of press moulds and further fabricating the base of solar cells using the mixture of recyclable materials and binding compounds, it is necessary to build three-dimensional model of solar tiles in SolidWorks (Fig. 2) - the software package of the computer-aided design system for automation of an industrial enterprise at the stage of design studies and process design of production operations, ensuring the fabrication of products of any complexity and purpose [5].

After the development of the three-dimensional models and the fabrication of the press mould, the samples of bases of the designed concentrator solar tiles have been made in accordance with the drawings and models presented at Fig. 1 (to the right) and Fig. 2. Due to the versatility of the developed model, the concentrator type of solar types by some manipulations can be transformed into planar tiles, which at the initial and preliminary stage of designing and research is an important

**Fig. 3. Planar and concentrator solar tiles made and assembled in accordance with the developed model**

achievement necessary for comparing various parameters of these two types of solar tiles (Fig. 3).

If considered in detail, solar tiles comprise a bearing foundation with installed solar panel on the basis of semiconductor photovoltaic converter with an electric current-collecting cable. Solar panel is installed on the flat platform sunk in relation to the upper surface of the foundation, and is protected by hardening sealing compound. A solar panel integrated into tiles, comprises commutated silicon solar cells with 156-156 mm or 125-125 mm dimensions. It has protective coating of tempered glass and different voltage and electric capacity in dependence on the number of commutated solar cells. Fig. 3 in the middle part presents tiles with two standard cells with 125-125 mm dimensions, which simplifies commutation. A solar module integrated into tiles, with maximum filling of photoreceptive surface has also been fabricated (Fig. 3). Solar cells are sealed in a glass unit with two-component polysiloxane compound increasing their useful life period [2, 3].

Alongside with the planar version of solar modules in tiles, protected linear focusing cylindrical concentrators with the solar cells in the form of a band in its focus (Fig. 3 to the left), have been fabricated and applied. The use of concentrators in roof solar panels allows to reduce semiconductor material consumption, as well as the cost of the module itself [6, 7]. The case of roof solar panel can be made of high impact plastic, ceramics or mixture of sand and reworked plastic. The internal cavities of the case are molded to locate protective coating for semicylindrical parabolic mirror reflectors, an optical deflection system and receivers made of commutated solar cells. Protective coating can also be made in the form of an optical deflection system, and mirror reflectors of flat mirror facets whose planes are parallel to the focal axis, and the width of glass mirror facets in the meridian plane is equal or exceeds the width of the receiver composed of commutated solar cells thus providing even illumination of photoreceivers and preventing overheating of their local spots. Semicylindrical parabolic mirror reflectors can also be made of polished aluminium alloy sheet. The side walls of internal mold cavities to place semicylindrical parabolic mirror reflectors are located in the meridian plane and are equipped with mirror reflecting coating. In the tile case there are slots perpendicular to the plane of protective coating passing through the focal axes of semicylindrical parabolic mirror reflectors, where additional protective glass or transparent plastic covering is installed on sealant, and photoreceivers composed of commutated solar cells are installed between additional protective covering and the case. Every space between additional protective covering and the case, where photoreceivers composed of commutated solar cells are placed, is filled with transparent polysiloxane compound prolonging solar cells service life. Inside the case there are cavities for cable connection of receivers made of commutated solar cells to the junction box installed in the case cavity between the semicylindrical parabolic mirror reflectors on the other side of the roof solar panel, and equipped with a conducting cable for commutation with other roof solar panels.

In addition, to increase work throughout the day, concentrator solar tiles comprise an optical deflection system made of a multitude of prisms oriented in one direction, with a sharp angle between the surface of solar ray inlet and the surface of solar ray outlet [6, 7].

Roof concentrator solar panels are installed at the southern slope of a roof at two possible angles – for maximal generation in summer months and for maximal generation in winter months. The use of an additional prism optical deflection system increases an effective aperture angle of the concentrator solar module and the period of work in stationary mode, and reduces cosine losses. The roof concentrator solar panel works in stationary mode without sun tracking and collects direct and diffuse solar radiation at the receiver within the aperture angle.

Alongside with concentrator solar panels it is possible to use planar roof solar panels which are installed at the southern slope of a roof at an angle optimal in terms of annual electric power in a geographic region where solar tiles are installed. Solar tiles of this type are more suitable for the use in regions where overcast days and days with low solar radiation prevail, whereas concentrator solar panels are more suitable for regions with high indicators of direct solar radiation (southern, Siberian and mountainous regions, clean atmosphere).

As roof solar panels combine such functions as roof construction, electricity generation and environmental improvement due to the use of recyclable materials in its production, they are practically an ideal solution of the problem of electricity supply to private houses on the basis of independent solar power generation. Alongside with the abovementioned functions, solar tiles are characterized by very attractive and esthetic design in contrast to standard planar solar module whose rather unattractive exterior was one of the reasons for which the most demanding house owners are reluctant to buy and install them.

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# РАЗРАБОТКА КРОВЕЛЬНОЙ СОЛНЕЧНОЙ ПАНЕЛИ

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В статье предложен электрогенерирующий покрывной материал в виде черепицы, изготовленной из вторичного сырья и имеющей в основе солнечные элементы и концентратор солнечного излучения. Рассмотрены два варианта изготовления солнечной черепицы – бесконцентраторное исполнение с планарными солнечными элементами, гелем, увеличивающим срок службы элементов, и концентраторное исполнение с нарезанными солнечными элементами, гелем, концентратором солнечного излучения и оптической отклоняющей системой в виде призмы. Наряду со строительным значением солнечная черепица позволяет решить задачи электрогенерации, увеличения срока службы солнечных модулей и вторичного использования пластикового вторсырья.

**Ключевые слова:** электроснабжение, солнечная черепица, концентратор солнечного излучения, полисилоксановый компаунд, вторичное сырье.

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# ANALYSIS METHODS OF NONEQUILIBRIUM THERMODYNAMICS OF NATURAL EVOLUTIONARY PROCESSES

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*The thermodynamic bases of biological evolution are considered. The principle of hierarchical life development and interdependency of entropy and information in the evolution process is emphasized. Biophysical studies, including evolutionary processes, are carried out by a method of no equilibrium thermodynamics. Evolution of life towards more organized forms is process of step increase in total entropy.*

***Keywords:*** *distribution function; entropy; evolution; Onsager relations; Curie principle; free energy; genetic information; hierarchical system.*

In functional and morphological terms, living beings are the most complicated and highly-organized of all natural objects. It should be mentioned that in terms of thermodynamics, living systems, including at the cellular level, behave as no equilibrium objects with strong in homogeneities.

The evolution of these systems is affected by changing of some parameters under the influence of external factors that may be termed governing factors. For equilibrium state, in most simple case the evolution of the system parameters is described by the relation [1]:

$$F_i(\{X_s\}, \lambda) = 0, \qquad (1)$$

where $F_i$ – arbitrary complicated function depending on macroscopic variables of the system and parameters λ.

This relation is true under specific limitations. For example, the laws of evolution should be such as to meet the requirements of positivity of temperature and chemical concentration characteristic of this system.

A possible state of a system is determined by the physical parameter *K*. The value *W* may be a complicated function of criteria and conditions.

A possible state of the system is characterized by the distribution $f(x, t)$:

$$S(t) = -K\int f(x,t)\ln f(x,t)dx. \qquad (2)$$

In addition, depending on specific type of *K* and the character of averaging of the written $f(x, t)$. the expression (2) can describe entropy by Bolzmann, Shannon and Gibbs formulas. Particularly, on this basis in [2] the corresponding distribution function for entropy is introduced.

The definition of Boltzmann information (entropy) is always related to the variational problem of determination of maximal probabilities.

For nonisolated systems exchanging energy and substance with environment, changing of entropy constitutes a sum of two members. One of them - $d_eS$ is conditioned by processes in the system [2]:

$$\frac{dS}{dt} = \frac{d_iS}{dt} + \frac{d_eS}{dt}. \qquad (3)$$

For an isolated system $d_eS = 0$, the equation (3) is reduced to the dependence $dS = d_iS = \geq 0$.

For irreversible processes, the phenomenological relations of irreversible processes are used (Onsager relations) as rheological equations [4]:

$$I_k = \sum_{k=1}^{N} L_{kl} X_l; (k = 1, 2, ..., N), \qquad (4)$$

where *N* – the number of independent physical processes; $L_{kl}$ – the matrix of phenomenological (kinetic) coefficients connecting the flows $I_k$ and thermodynamic forces $X_l$.

In general, flows and thermodynamic forces are tensor quantities of any rank. Physical meaning of kinetic coefficients can be determined within the framework of molecular-kinetic theory. The number of kinetic coefficients different from zero in (4) are limited by the Curie principle, in accordance with which components (components of vectors along the coordinate axes are meant) of flows will depend not on all the components of thermodynamic forces. For instance, in the case of an isolated system processes of various tensor dimensions do not interact with one another. Moreover, within the framework of Onsager relations, the relations of Onsager-

Casimir are taken as independent postulate – the so-called principle of reversibility):

$$L_{l;} = (B,\Omega) = \varepsilon_k \varepsilon_l L_{lk(-B,-\Omega)} . \qquad (5)$$

Here $B$ is magnetic induction, $\Omega$ – angular spin rate of the system, and $\varepsilon_k = 1$ for even (energy, concentration) and и $\varepsilon_l = -1$ for uneven (momentum density) macroscopic parameters (even or uneven functions of particles velocity). For an isotropic, not rotating system, in the absence of external magnetic field, the relations of the symmetry (5) assume a more simple form:

$$L_{kl} = L_{lk} , \qquad (6)$$

where $L_{kl}$ - scalar values.

In dependence on the system characteristic, the value $d_eS$ can be both positive and negative. If $d_eS$ is negative and exceeds $d_iS$, in absolute value, some stage of evolution can proceed with general lowering of entropy:

$$\frac{dS}{dt} \angle 0 . \qquad (7)$$

This means that in the course of evolution orderliness decreases at the cost of entropy outflow.

The distinction of energy of life from technical thermal processes is explained by the fact that it uses electrochemical thermodynamic cycles. In this process living systems use not internal energy but free energy - Helmholtz thermodynamic potential.

A key peculiarity of free energy is that entropy $S$ does not rank among independent task variables. This means that interaction of energy and the amount of extraneous information is not fundamental for the process of life and its evolution. For genetic mechanisms the crucial thing are substance properties and, consequently, the description of the process of independent variable should include chemical potential. In living systems the production of energy prevail over its dissipation, and excess energy is accumulated [6].

Life is principally related to hierarchic synthesis of randomness and for each subsequent hierarchy level specific conditions are synthesized differing from the conditions at preceding stages. In the result, entropy describing a number of probable states at a preceding level of hierarchy is supplemented by entropy generated by new states regulating new conditions. Therefore, the emergence of life is the process of hierarchic increase of communal entropy. The hierarchy of information synthesis is the evolution of life [8]. Entropy of evolution of any species of living beings is written in the following way:

$$S_{к} = S_{к,ген} + S_{к,сам} , \qquad (8)$$

where $S_{к,ген}$ – the sum of measure of genetic information; $S_{к,сам}$– measure of information on self-organization processes for which the properties of the system are established by the value $S_{к,ген}$.

For living beings whose development was determined by selection, in the right part of equation $S_{к,сел}$ should be added – the measure of information arising from selection activities.

For evolution of life, the relation of the amount of genetic information for two successive levels of the hierarchy of information synthesis, is of importance. Let us term the number $Z_k$, the value of genetic information, It shows how many times the information amount will decrease in the process of transition to the following level of the hierarchy of information synthesis:

$$Z_k = \frac{S_{(k+1),ген}}{S_{k,ген}} . \qquad (9)$$

Excess of dissipation over energy output, even if possible, can occur only as occasional incidence. However, at the stages of life birth, the principle of development appears, providing excess of energy output in comparison with demand in it. Concentration of energy in this process is used as the basis for genetic information synthesis.

The processes of evolution are observed in a whole variety of dissipative structures. One of examples is the process of biological population. The problem of population dynamics is solved by the method of boundary cycles [9]. An example of evolution is the dynamics of processes characteristic of any ecosystems, including natural environment. The laws of evolution of self-organization of any language are also worth noticing [10]. Evolutionary processes can also be illustrated by the activities of social and ecological systems.

In nature, development occurs on the basis of chain hierarchy: randomness-conditions-memorization. A synonym of memorization in nature is stability of its objects and processes. This results in continuous development, in the process of which entropy grows but stepwise in a hierarchical way.

**Conclusion**

Research of biophysical processes, including evolutionary ones, is carried out by the method of nonequilibrium thermodynamics. It is pointed out that the hierarchic character of information synthesis in the course of the evolution and development of living beings, is described in terms of entropy. The evolution of life towards more organized forms is the process of stepwise increase of communal entropy.

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## АНАЛИЗ МЕТОДОВ НЕРАВНОВЕСНОЙ ТЕРМОДИНАМИКИ ПРИРОДНЫХ ЭВОЛЮЦИОННЫХ ПРОЦЕССОВ


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Излагаются термодинамические основы биологической эволюции. Подчеркивается принцип иерархического развития жизни и взаимозависимость энтропии и информации в процессе эволюции. Исследования биофизических, в том числе эволюционных процессов проводится методом неравновесной термодинамики. Эволюция жизни в сторону более организованных форм есть процесс ступенчатого увеличения суммарной энтропии.



**Ключевые слова:** функция распределения; энтропия; эволюция; соотношение Онсагера; принцип Кюри; свободная энергия; генетическая информация; иерархическая система.


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# ON THE FUNDAMENTAL PRINCIPLE AND LAW OF NATURE

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*In nonequilibrium thermodynamics and energetics the second law of thermodynamics is considered fundamental. In quantum physics and the theory of relativity such outstanding scientists as M. Planck, H. Poincaré, A. Einstein accepted the Hamilton principle of least action as a fundamental law. Since the latter half of the X1X it has been commonly believed that evolution is governed by the second law of thermodynamics. This approach has given rise to certain problems: the heat death of the Earth and Universe, the contradiction between evolution in accordance with the second law and the theory of biological evolution, as well as other scientific problems. The author has solved these problems, revealing the law of survival whose essence is contrary to the essence of the second law. The law of survival and the second law in the form of reflectional dynamic symmetry constitute the general principle of natural science – the principle of energy extremality of self-organization and progressive evolution. The axiom simultaneously reflecting the law, the origin and the principle «life-death» has been substantiated. The law of survival directs progressive evolution and allows to explain the birth and functioning of self-regulating systems. The second law destroys the structures of systems that have left self-organized state, and provides the turnover of substances participating in evolution, which has been confirmed by empirical data of evolution and biosphere functioning. .*

***Keywords:*** *evolution of biosphere, the second law of thermodynamic, the law of survival, the fundamental law, the principle of energy extremality of self-organization, progressive evolution, turnover of substances.*

**State-of-the-art**

Postnonclassical (evolutionary) paradigm of cognition takes into account the peculiarities of the process of evolution of self-organizing nature in further development of cognition. The fundamental generally recognized fundamental principle of self-organization is the synergetic principle of dependence, whose essence is the reduction of variables of a system being analyzed to consider only one of them – the variable of order. Unfortunately, this principle does not reflect important peculiarities of the essence of self-organization phenomenon. All branches of physics assume the conservation laws as fundamental. In classical thermodynamics and energetic the second law of thermodynamics is commonly considered a fundamental law, and in quantum physics and the relativity theory, according to some outstanding scientists (M. Planck, H. Poincaré, A. Einstein), the Hamilton principle of least action is recognized as a fundamental law. The second law of thermodynamics is not reflected in the theory of relativity and quantum physics. Evidently, for this reason they faced the problem of negative energy.

The conservation laws cannot reflect the peculiarities of the evolution process. All the definitions of the second law of thermodynamics are expressed in a restrictive form, and its basic function – entropy – has destructive quality. The second law can be applied exclusively to equilibrium (not self-organizing) systems. Only self-organizing (nonequilibrium) systems not subjected to the second law, have the ability to evolve. The Hamilton principle of least action in its essence is an implicitly expressed principle of extremal action substantiated by L. Euler who relied on the Maupertius principle of least action. The principle of extremal action was also substantiated by G.S. Landsberg [1, p. 201] on the basis of the Fermat principle. The principle of extremal action is symmetrical and has both maxima and minima. This refers equally to the Hamilton principle of least action. The principles of least action as formulated by Maupertius and Ferma, are asymmetrical and essentially applied only to self-organizing (nonequilibrium) systems. The second law is also asymmetrical. It is applied only to equilibrium (not self-organizing) systems.

Physicists have revealed that evidently for this reason, as soon as the second law was formulated it has been in conflict with «all the other branches of physics». In this connection, Poincare substantiated the recurrence theorem wherein the impossibility of the existence of a law whose function would grow continuously and unidirectionally, like entropy, was revealed. Considering the process of evolution, logically asymmetrical laws cannot be fundamental laws of nature, that can be explained by the following: in the process of progressive evolution, biophil substances involved therein, undergo two radically different states – self-organized (nonequilibrium, live) and equilibrium (not self-organized, conservative, chaotic). This postulate is confirmed by empirical facts of the Earth biosphere evolution. Most evolutionists and researches of related branches of science consider the second law of thermodynamic a law directing progressive (global) evolution.

**The real role of the second law of thermodynamics in evolution**

The Earth does not have regular exchange of substances with its environment. Our planet is a thermodynamically closed system in terms of substances exchange. Progressive biological evolution on the Earth is possible only in the presence of turnover of biophil substances. The presence of such turnover has been experimentally established. Only some tenths of one percent of the whole amount of biophil substances residing on the Earth surface - nitrogen [2] and carbon [3] are circulating in the living part of the biosphere in a closed cycles. This fact testifies to high energy and substance economy of evolution process. For turnover of biophil substances, universal and continuous destruction of systems that have left self-organized state is required. When a system leaves self-organized state, its structure becomes equilibrium and subject to the second law. It is no coincidence that its fundamental function – entropy has destructive quality. It acts everywhere and continuously destroy such structures, releasing biophil substances in the form suitable for recirculation in self-organizing systems. At the same time, systems in self-organized state, continue to function adequately and to emerge again in correspondence with the law whose essence is opposite to the second law of thermodynamics.

**How the second law of thermodynamics came to be recognized as the law directing evolution**

The value (term) «entropy» and the original wording of the second law of thermodynamics were introduced in 1850 by German physicist Rudolf Julius Emanuel Clausius: Эммануил [ quoted after 4, v. 2, p 218]: The second law, in the form which I have given it, states the fact that all transformations which occur in nature occur in a certain sense which I have taken as positive, of themselves, that is, without compensation, but that they can only occur in the opposite or negative sense in such a way that they are compensated by positive transformations which occur at the same time. The application of this law to the universe leads to a conclusion to which W. Thomson first called attention and about which I have already spoken in a recently published paper. This conclusion is that if among all the changes of state which occur in the universe the transformations in one sense exceed in magnitude those in the opposite sense, then the general condition of the universe will change more and more in the former sense, and the universe will thus persistently approach a final state».

Considering the applicability of the first and the second laws of thermodynamics to the Universe, Clausius gives the following short wording of these two postulates: 1. The energy of the universe is constant; 2. The entropy of the universe tends toward a maximum. It is in this way that the problem of the «heat death of the Earth and the Universe» was brought about and caused so many controversies in science, philosophy and sociology.

This problem is in gross contradiction with the theory of biological evolution in accordance with which the structures and functions of living organisms are progressively developing and their entropy does not grow but decreases. Such interpretation of the second law of thermodynamics leads to its inconformity (contradiction) with all the other branches of classical physics. The above substantiation of the problem of the heat death of the Earth and the Universe elaborated by Clausius on the basis of heat definition of entropy, appears to be inconsistent. The theoretical and empirical data of the day and even current data on energy processes of the Earth and, even more so, of the Universe are rather scarce and clearly insufficient for such ambitious generalization. Such generalization is not plausible without regard to conditions and causes of emergence and functioning of the animated nature on the Earth. It is surprising that scientists of the day and of succeeding generations could take seriously the idea of the heat death and the governing role of the second law of thermodynamics in evolution.

**The substantiation of the statistic definition of entropy as manifestation of disbelief in the possibility to extend the application of this value to the whole nature in the Clausius' substantiation of thermal processes**

Ludwig Boltzmann revealed the statistical basis of the second law of thermodynamics. He evidently thought that the Clausius thermal definition of entropy can be extended only to equilibrium systems and cannot be applied to self-organizing nonequilibrium (living) systems. He admired the theory of biological evolution by Ch. Darwin and dreamt to extend a similar theory to the whole nature, probably with the use of the statistical definition of entropy. In the result obtained from such definition, in the right part of the formula unexpectedly the negative sign appeared (H = – KlnW). This caused discussion and aroused disbelief in correctness of the formula construction. Boltzmann termed this formula «H-function». In the course of discussion on this formula, an adequate opinion was offered that the formula of the statistical determination of entropy in the right part must have the two signs: both negative and positive. The analysis shows [5] that the presence of the negative sign in the right part of the formula is conditioned by the following: alongside with the second law of thermodynamics, in the construction of the formula Boltzmann used the postulates of classical mechanics wherein the essence of the Maupertius principle of least action was implicitly present. After Boltzmann's death, M. Planck without any explanation transformed this formula into the following: S = KlnW. He termed it the Boltzman formula for statistical determination of entropy [6]. However, this formula, as well as the Clausius formula for thermal determination of entropy determine this value only for the case of equilibrium (not self-organizing) systems.

As demonstrated both theoretically and experimentally in the works by theoretical physicist Yu. L. Klimontovich, in self-organization processes, entropy does not grow but decreases [7], consequently, the formula for statistical determination of entropy as extended to the whole nature, in its right part must have the two signs; positive (for equilibrium systems) and negative (for non-equilibrium, self-organizing systems).

**The second documented case of disbelief in the second law of thermodynamics as the fundamental law of nature is the statement by K.A. Timiryazev in his Croonian lecture «The Cosmic Role of Green Plants»**, delivered on April 30, 1903 at the session of the Royal Society of London [8]. It is impossible to explain the main energy process of the Earth biosphere – photosynthesis – on the basis of the second law of thermodynamics. This process is not in keeping with the definition of entropy and clearly contradicts the problem of the heat death. Timiryazev was admitted as a member of the Royal Society of London only in 1912. This demonstrates that the members of this society were not able to give an adequate answer to the challenge to fundamental science contained in the title of his lecture at the moment of its delivery.

In fact, answer to the Timiryazev's challenge was presented in A. Einstein work «On a Heuristic Viewpoint Concerning the Production and Transformation of Light" written in 1902-1905 and published in 2005 [9]. For this work the author was awarded the Nobel prize. It presents the substantiation of quantum transformation of radiation energy – photoeffect. Radiation energy is not only generated (emitted) but transformed by whole photons (quanta). Photoeffect (physical, chemical, biological) is proportional not to the amount of absorbed energy but to the amount of effectively absorbed photons. Direct transformation of radiation energy through photosynthesis of plants and photovoltaic cells can be explained by the law of quantum equivalence and not by the second law of thermodynamics. This can be considered as the evidence of Einstein's recognition of inadequacy of the second as a fundamental law of nature.

However, conceptually, in terms of photoeffect, radiation exergy (free energy at the inlet to a converter) can be determined only for cases when a radiator and a radiation converter have spectral characteristics similar to those of the absolutely black body. Spectral characteristics of green leaves of plants and photovoltaic converters differ fundamentally from the corresponding characteristics of the absolutely black body. The methodology of determination of radiation exergy for plant cultivation has been defined by the semi-empirical method and discussed at the 3rd International Conference on radiation transformation in plants. The results were published in the scientific magazine of the Humboldt university [10]. To regulate the determination of this value domestic industry standards have been developed. In Germany the determination of this value is regulated by national norms. The methodology of determination of radiation exergy for plant cultivation has made it possible to develop the theory of optimization of agricultural technologies of harvesting [11].

### The law of survival whose essence is opposite to the essence of the second law of thermodynamics

In the discussions on the problems of the second law of thermodynamics, some scientists – V.I. Vernadskiy, H.Helmholtz, K.A. Timiryazev, N.A. Umov, K.E. Tsiolokovskiy put forward the hypothesis of the existence of yet not discovered law whose essence is opposite to the essence of the second law of thermodynamics. The author of this work believed in this hypothesis and substantiated such a law termed «the law of survival». Its essence is as follows [11]: any element of a self-organizing

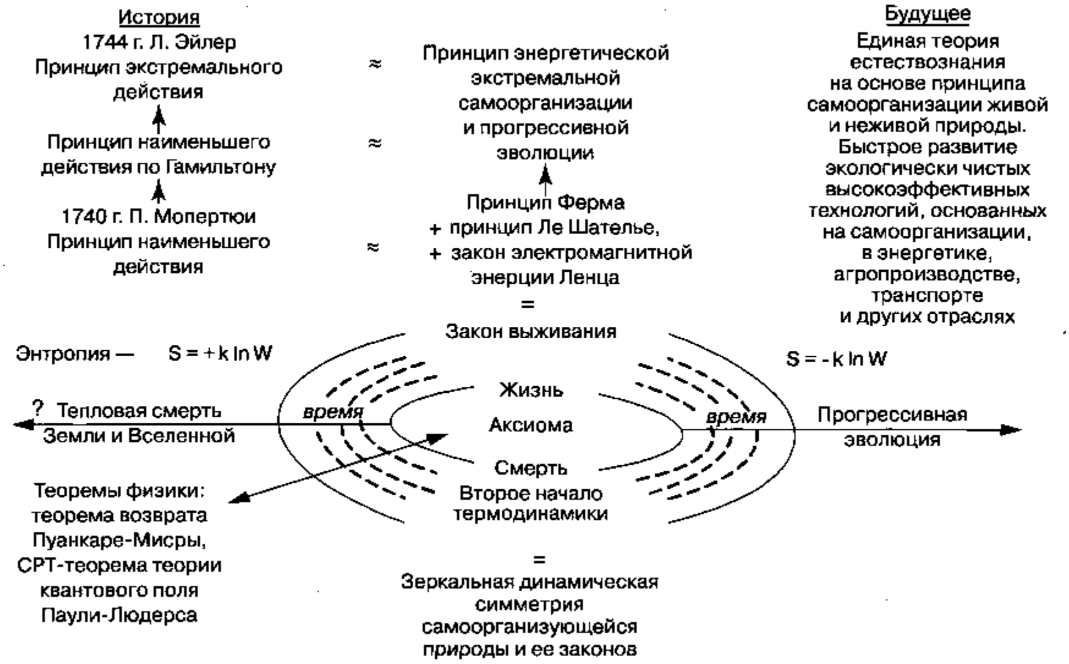


**Fig. 1. The logical scheme of connection of fundamental essence of the law of survival, the second law and the principle of energy extremality of self-organization with the axiom of life and death, phenomenal physicochemical principles and fundamental theorems of physics**

nature in its development (individual, evolutionary) spontaneously tends to the state of the most complete (effective) use of available free energy by a system of the trophic level whose part it forms. The analytic expressions of the law of survival are similar to those of the second law of thermodynamics, but their right part have the negative sign [11]. The law of survival and the second law are united in the form of reflective dynamic symmetry into the general natural principle of energy extremality of self-organization and progressive evolution. The application of the second law is limited only to equilibrium (not self-organizing) systems. Self-organizing natural systems come to life and function in correspondence with the law of survival. The axiom has been revealed that encompasses the law of survival, the second law of thermodynamics and the principle of energy extremality of self-organization. This axiom is as follows: the phenomenon of the birth of any object, some period of its existence (life) and its destruction (death). In short: life – death. Logical link of this axiom with the law of survival, the second law of thermodynamics and the principle of energy extremality, as well as with the fundamental theorems of physics and extremal phenomenal principles is presented at Fig.1.

In nature survival law is manifested in the form of phenomena which are reliably established empirically or mathematically but have not yet been explained in scientific terms. They include: phase transfer, golden proportion, fractal structures, solitons, ontogeny or biogenetic law, high ability of all species without any exception for reproduction, as well as phenomenal extremal principles: the Fermat principle, the principle of least work, Le Chatelier's principle, and the Lenz law (rule) of electromagnetic inertia. All phenomena are self-organizing. The law of survival has made it possible to explain their fundamental positive essence – energy and substance economy, that in itself confirms the relevance of this law.

On the basis of the law of survival and the principle of energy extremity of self-organization it became possible to solve the age-old scientific problems caused by classical thermodynamics [11], to solve the major problem of biophysics – the integration of the theories of classical physics with biology. This also confirms the relevance of considered advanced scientific ideas and their high cognitive value. It has been shown that they are consistent with the K. Marx's immanent law of method reversal [12]. With the use of the law of survival and the principle of energy extremality of self-organization, it became possible to reveal really ideal properties of progressive evolution, its spontaneous tendency to economy: energy, substance and informational, which results in beauty and harmony of evolving natural objects. As they grow in complexity, the evolution process is accelerated [13]. These physically nonseparable really ideal properties are confirmed both in theoretical and experimental terms. For example, acceleration of evolution as an evolving object grows in complexity, is confirmed by the data of experimental determination of structures of the whole genomes of organisms. Of the structures of the human whole genome, approximately 10% differ from the structures of the whole genome of mice, whereas only one percent of the structures of the whole genome of mice differ from the structures of the human whole genome. This data demonstrate that since the time of separation of genetic lines of humans and mice, the structures of the human genome as a more complex object of evolution, developed ten times faster than the structures of the mice genome.

The reality of existence of the law of survival and the principle of energy extremality of self-organization is confirmed by the following peculiarity of progressive evolution of biosphere in the Earth conditions. The Earth does not have regular exchange of substances with its environment. It can be considered as a thermodynamically closed system in terms of exchange of substances. Therefore, progressive evolution thereon is possible only in the context of turnover of substances involved in evolution. Such turnover of biophil substances has been experimentally proved. Only some parts of one percent of the total amount of biophil elements existing on the Earth surface – nitrogen [2] and carbon [3] are circulating in the live part of the biosphere in a closed cycle. This is the evidence of high energy and substance economy of evolution in the Earth conditions. To provide such circulation it is necessary to destroy the structures of objects that have left self-organizing state (dead organisms). Entropy has such destructive quality. Being the fundamental function of the second law of thermodynamics, it «universally and continuously grows» destroying structures. It should be added that it is referred to the structures of objects that have become equilibrium after they have left the self-organized state. In accordance with the law of survival, self-organized objects continue to function and again come to life, using substances of the destroyed structures. This vividly demonstrates the inconsistency of the fear of the heat death of the Earth and the Universe. In recent years scientific methodologists in cognition attach great importance to the global process of self-organizing nature – its global (progressive) evolution [14]. Only postnonclassical (evolutionary) paradigm of cognition, considering evolutionary determinism, allows to integrate the human science with other natural sciences. In the antique period, the main tool of cognition was visual observation of material objects of environment – the product of progressive evolution. However, the obtained results, for example, such as the Plato's holon concept, even nowadays are not yet fully apprehended in terms of cognition. In accordance with this concept, it is reasonable to consider really ideal nonseparable properties of progressive evolution, revealed on the basis of the law of survival and the principle of energy extremality of self-organization, as a holon of the highest level of self-organizing nature. Physical constants are Plato's holons – entities of lower level of self-organizing Nature. tl of self-organizing nature. Holons of lower level of self-organizing Nature are the quantum of action – the Planck's constant and other physical constants.

An important achievement of science in the XXth century is the substantiation of the anthropic cosmological principle [15]. It has several wordings reflecting its essence not exactly enough, which provokes cautious attitude on the part of science methodologists and theoretical physicists. The fundamental essence of this principle is teleological: it demonstrates that at the very early stage of progressive evolution – the emergence of microparticles characterized by physical constants, the birth of human observer was envisaged.

It is possible to understand this essence only on the basis of progressive evolution which gave birth to this principle. We have made an attempt to formulate such wording of the anthropic cosmological principle that would demonstrate its scientific validity. At the macrolevel, it is as follows: any general scientific theory is not correct if it does not envisage the emergence of conditions for the birth of life and the human observer. At the level of microparticles it is defined as follows: in estimating the conditions for birth of life and the human observer, physical constants used in these calculations, should be determined to the accuracy of no less than 9-12 decimal places. The both wordings reflect the objective reality satisfying the requirements of scientific validity, in this case, of the anthropic cosmological principle [16]. It should be considered as a direct consequence of progressive evolution and the level of its precision.

For anthropology, in general, this principle is of the same importance as the law of gravitation for physics and engineering. It is the source of theorization of the anthropology and sustainable development of human society as an integral part of progressively developing unconscious nature. The understanding of the essence of progressive evolution on the Earth (in the thermodynamic system closed in terms of exchange of substances) as substances periodical passing through two principally different states: self-organized (nonequilibrium) and equilibrium (chaotic), allows to exactly define the role of the second law of thermodynamics in evolution as the utilizer of the structures of objects that have left the self-organized state.

This confirms the objectivity of the existence of the law of survival and the principle of energy extremality of self-organization, as well as the governing role of the law of survival in the process of progressive evolution of the whole self-organizing nature. Considering the properties of progressive evolution and the anthropic cosmological principle, it is possible to accelerate the determination of the essence of the conscious man and his role in the phylogenesis of the whole nature. At the current level of cognition it can be said without prejudice that the development of human society should be in agreement with the development of unconscious part of nature. The history of human civilizations testifies to the fact that in general their development was governed by religious doctrines or was quite spontaneous.

In 1977, having realized the invalidity of application of the second law of thermodynamics to self-organizing natural systems, the author of this article began to develop the exergy theory of yield which was completed by 2007 [11]. It allows to develop computer systems for energy and resource-saving crop production [11], which are in high demand all over the world. On the basis of the law of survival and the principle of energy extremality of self-organization, age-old scientific problems caused by classical thermodynamics have been solved, modern physical theories and biological knowledge have been logically integrated, really ideal properties of progressive evolution have been revealed and the scientific validity of the anthropic cosmological principle have been substantiated. This principle appears to be the source of the theoretization of sociology and anthropology in general, as well as the theoretical basis of social justice and sustainable development of human society and the unconscious nature. It has been demonstrated that on the basis of the survival law the current global problems – energy, food and environmental – can be solved [17].

Supplementation of generally recognized scientific ideas with the law of survival and the principle of energy extremality of self-organization allows to establish scientific foundations of comprehensive integration of knowledge without invoking the religious teachings. The use of these scientific innovations accelerates the development of technologies and technical means: from biotechnologies to information and communication technologies in modern politics. These provisions are used in the first manual on agricultural biotechnology [18] and in the program «Information and communication technologies in modern politics» designed for certification of academic teaching employees of higher educational institutions by the training center of the Russian Academy of State Service at the President of the Russian Federation [19]. The fundamentals of consolidated knowledge will make it possible to logically formalize empirical knowledge and integrate it with the general system of science.

We believe that at the current stage of scientific development the law of survival and the principle of energy extremality of self-organization of progressive evolution are the fundamental law and the principle of nature.

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## О ГЛАВНЫХ ПРИНЦИПЕ И ЗАКОНЕ ПРИРОДЫ

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В равновесной термодинамике и энергетике главным законом считают второе начало термодинамики. В квантовой физике и теории относительности выдающиеся ученые: М. Планк, А.Пуанкаре, А. Эйнштейн за главный закон принимали принцип наименьшего действия в форме Гамильтона. Со второй половины XIX столетия принято считать, что эволюцию направляет второе начало. Это вызвало проблемы: тепловой смерти Земли и Вселенной, противоречие между эволюцией по второму началу и теорий биологической эволюции, а также иные проблемы науки. Автор решил эти проблемы, выявив закон выживания, сущность которого противоположна сущности второго начала. Закон выживания и второе начало в виде зеркальной динамической симметрии образуют общий принцип естествознания - принцип энергетической экстремальности самоорганизации и прогрессивной эволюции. Обоснована аксиома одновременно отображающая: закон, начало и принцип – «жизнь – смерть». Закон выживания направляет прогрессивную эволюцию, позволяет объяснить возникновение и функционирование самоорганизующихся систем. Второе начало разрушает структуры вышедших из самоор-

ганизованного состояния систем и обеспечивает круговорот веществ, участвующих в эволюции. Эмпирические данные эволюции и функционирования биосферы подтверждают это.

**Ключевые слова:** эволюция биосферы, второе начало термодинамики, закон выживания, закон главный, принцип энергетической экстремальности самоорганизации, прогрессивная эволюция, круговорот веществ, прогрессивна эволюция.

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# THE LOGICAL-MATHEMATICAL ANALYSIS FOR SUBSTANTIATION OF EFFICIENCY OF HEAT PUMPS AND REFRIGERATORS

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*The possibility of exergy analysis of conversion of ambient heat used by heat pumps (HP) and refrigerating machines (RM) - the most effective heat and cold generators - has been revealed and its methods have been developed. On the basis of logical-mathematical analysis the relevance of the use of the terms of thermodynamic and exergy efficiency not only in relation to heat engines but also in relation to HP and RM has been substantiated. The lack of substantiation of the use of thermodynamical and exergy efficiency values in relation to HP casted doubt on high efficiency of electric power conversion into low-temperature heat energy. This has led to RF lagging in the HP production and use. Of the total amount of fuel annually consumed in RF, over 50% are used to generate low-temperature heat. Exergy of this part of fuel is not used. This is one of the reasons of high energy consumption of agricultural goods and RF gross domestic product. It is urgent to modernize energy supply to RF agricultural complex and housing and utility sector: to replace conventional boilers by mini co-generation plants using HP.*

**Keywords:** *refrigerator, heat pump, exergy analysis, thermodynamic efficiency, energy consumption, mini co-generation plants, co-generation.*

The heat pump (HP) was invented in 1852 by outstanding English physicist William Thomson (Kelvin) and was termed «thermodynamic heating». HP started to be implemented in practice in the 1930-ies. It should be mentioned that adequate explanation of the operating principle of HP and RM on the basis of the second law of thermodynamics is complicated in principle. In various manuals and other technical literature their operation is explained by «inverse Carnot cycle» which differs from direct Carnot cycle in that it proceeds not «clockwise but counterwise». Such explanation of cannot fully reflect the energy converting efficiency of HP and RM. In fact, in HP and RM self-organizing energy-saving natural process is used, namely, phase transition of heat carrier: evaporation – condensation, allowing to transfer heat from less warm body to more warm one. To characterize energy converting efficiency of HP and RM not efficiency indicators are used but «heating» and «refrigerating» coefficients correspondingly.

HP came to be used widely in the late 1960-ies because of the first manifestation of the world energy crisis. According to scientific literature reviewed [1] the use of advanced reverse-cycle heat pumps to generate low-temperature heat (heating, air-conditioning, hot water supply) makes it possible to reduce fuel consumption 3 times (for air-air HP) and 5 times (for air-water HP) in comparison with the use of conventional fuel heat generators. Unfortunately, so far in the Russian Federation HP have not been widely used and instead, conventional boilers are used and built. Exergy of fuel consumed in boilers and other fuel heat generators not producing electricity, is practically not used. Of total amount of fuel annually consumed in RF, nearly 50 % is used in such inefficient way. This is one of the reasons of high energy intensity of RF gross domestic product (GDP) and high cost of housing and utility sector. It is by accident that power engineer A.B. Bogdanov suggested to consider the «boilerization» of Russia as «national disaster» [2]. [2]. In the world energy intensity of products (services) and GDP is considered as main indicator of scientific and technological and socio-economic progress of an industry, a country or a region. In accordance with the data of the International Energy Agency (IEA) in 2006 the GDP of Japan had the lowest energy intensity. If we take the indicator of the Japanese GDP as a unit, the index of energy efficiency of fuel use in other countries was as follows: Great Britain – 1.4; Germany – 1.75; France – 1.9; the USA – 2.1; India – 8.0; China – 9.0; Russia – 18.1. Over the past period of time there have not been any positive changes in RF domestic energy industry. In most foreign countries energy industry is improved due to the considerable expansion of the use of renewable energy sources (RES), including ambient heat, by means of HP application.

Alongside with ambient heat, the use of solar and biomass energy is also most successfully expanded. As it is known, the entropy method in principle is not suitable for analysis of these RES conversion. In the 1980-ies most energy experts started to use exergy analysis instead of entropy analysis, as it is more simple and reliable which allows to use it to analyze RES conversion. The basic works by Prof. V.M. Brodyanskiy, a leading expert in exergy analysis, such analysis for these RES conversion has not yet been developed [3, 4]. It is evident that this fact is the reason of RF lagging in the use of HP and other RES converters. This is confirmed by the experience of discussions on the necessity of the HP use. When it is mentioned that HP has high energy efficiency, opponents point out that HP «does not even has efficiency». So far the absence of reliable methods for determination of ambient heat exergy has not allowed to establish the values of thermodynamical and exergy efficiency for HP and RM.

The initial methodological basis for determination of ambient exergy and heat with the use of logical-mathematical method is the Carnot formula to determine thermodynamical efficiency ($\eta_T$) of a thermal machine:

$$\eta_T = 1 - \frac{T_0}{T_1}, \qquad (1)$$

where $T_1$ – temperature of heat carrier at the machine inlet; $T_0$ – the same at the machine outlet.

This indicator is generally recognized and widely used.

It should be noted that in accordance with this method, exergy, potential working efficiency of primary energy source used for heat generation in heat power machines ($e_T$) is determined through multiplying its total energy content (E) by thermodynamic efficiency.

$$e_T = E \cdot \left(1 - \frac{T_0}{T_1}\right) = E\eta_T . \qquad (2)$$

Extending the second law of thermodynamics to irreversible processes, R.J. Clausius noted that in the analysis of simple cyclic process of a thermal machine it is necessary to take into account two processes proceeding simultaneously therein: conversion of some amount of heat into work (or vice versa) and simultaneous heat transfer from more heated body to less heated body (or vice versa). He considered that between these two processes there is a certain connection which does not depend on the nature of a body (heat carrier) involved in cyclic process. In addition, he quite correctly pointed out that «the second law of thermodynamics must express relation between these two conversions». Further he proved that both conversions must be considered as phenomena of similar nature». He terms such two conversions that can replace each other «equivalent transformations». It should be considered that the both transformations proceed in opposite directions [5, v. 1, p. 212].

This Clausius conception in incipient form comprises a valuable idea for the solution of our problem – the substantiation of exergy analysis for ambient heat conversion and substantiation of relevance of the use of indicators of thermodynamic and exergy efficiency for HP and RM. By «two conversions» Clausius evidently means the following two processes: the process of heat conversion into work, proceeding in a thermal power machine – direct process, and the process of work conversion into heat proceeding in HP and RM – reverse process. These two mutually opposite processes can be considered as direct and reverse.

Usually direct process is characterized by the value of thermodynamic efficiency of the power machine ($\eta_T$), consequently, reverse process - the process of work conversion with the use of HOP and RM into heat, in accordance with the definition of inverse value in mathematical logic [6], can be characterized by inverse value to thermodynamic efficiency - $1/(\eta_T)$. With that in mind, thermodynamic efficiency of a heat pump and a refrigerating machine ($\eta_{ТН,ХМ}$) considering (1), can be determined by the following dependence:

$$\eta_{ТН,ХМ} = 1/\,\eta_T = T_1/T_1 - T_0\,. \qquad (3)$$

The dependence (3) is similar to the dependence for determination of «heating» and «refrigerating» coefficients for HP and RM, thus confirming the validity of the substantiation (3).

For further validation of the expression (3) let us consider the characteristics of advanced heat pumps available in the Internet [7]. In all Vitocal HP, whose characteristics are presented in Table 1, minimal inlet temperature amounts to +5 C$^{o}$, and maximal delivery temperature is +55 C$^{o}$ [7]. If we substitute these temperature values into the formula (3), we shall obtain the values of thermodynamic efficiency of the Vitocal heat pumps equal to 6.5, which are somewhat higher than the efficiency values given in Table 1 (5,6). The efficiency values given by the Vitocal company are evidently measured nominal values of real efficiency of heat pumps, as these two values coincide with relations of consumed electric power to heat output. In view of the above-mentioned, these values are not

**Table 1**

**Some characteristics of the Vitocal heat pump [7]**

| Characteristic | Heat pump type | | | |
|---|---|---|---|---|
| | **WW240** | **WW254** | **WW268** | **WW280** |
| Nominal heat capacity, kW (operating point W10/W35 in accordance with EN255) | 52.0 | 7.2 | 90,2 | 106.8 |
| Refrigerating capacity, kW | 42.8 | 60.0 | 74.0 | 87.8 |
| Consumed electric power, kW | 9.2 | 13.2 | 16.2 | 19.0 |
| Efficiency | 5.6 | 5.6 | 5.6 | 5.6 |

thermodynamic efficiency of heat pumps. Evidently, it is advisable to present them in technical documentation, alongside with nominal measured values of efficiency, as it was done by the Vitocal company.

The value of exergy efficiency of heat pumps can be determined by the value of HP real efficiency and its thermodynamic efficiency. It will be equal to 5.6/6,5 = 0.8. The value of exergy efficiency of any energy converter characterizes the measure of perfection of the given design of this type of converters.

Considering the value of exergy efficiency of the Vitocal reverse heat pump (0.8), it can be considered that this specific model is perfect enough for a reverse heat pump of the air-water type. It should be noted that without the determination of thermodynamic efficiency of any energy converter it is not possible to determine its exergy efficiency. The second independent method of the determination of thermodynamic efficiency of HP and RM is possible on the basis of the definition of the values of heat of phase transfer of energy carrier, which are basic working processes in these energy converters.

The peculiarity of the fundamental energy laws – the three laws of thermodynamics – is that their definitions have prohibitory character. This peculiarity is most vividly reflected in all existing definitions of the second law. Its initial definition: it is not possible for heat to flow from a colder body to a warmer body. Having taken into consideration the refrigerating machine work, lord Thomson-Kelvin invented «dynamic heating» and specified the initial definition of the second law, including the word «spontaneously» thereto. However, this refinement did not diminish the inferiority of the second law of thermodynamics as an independent law of nature. For example, it is evident from the general definition of HP efficiency given in the Great Soviet Encyclopedia, where it is noted that this indication can be greater than unity, as this converter uses not only electric energy but also ambient heat. HP real efficiency ($\eta_{TH}$) is empirically determined by the expression:

$$\eta_{\text{TH}} = \frac{Q_{\text{вых}}}{W_{э} + Q_{\text{TC}}}, \quad (4)$$

where $Q_{вых}$ – heat pumped by HP into heated premises; $W_э$ – electric energy consumed by HP; $Q_{TC}$ – ambient heat it uses.

Still heat $Q_{TC}$ can be theoretically determined only with the use of the Carnot formula (1), but as ambient heat does not have temperature gradient, the theoretical value of $Q_{TC}$ is equal to zero. Entropy of ambient heat is maximal, consequently, its potential convertibility (working capacity, exergy) is equal to zero. This conclusion on the basis of the second law of thermodynamics does not appear to be in agreement with practical results concerning HP high energy-converting efficiency. Their indicators of efficiency of electric energy conversion into low-temperature heat most commonly termed «heating coefficient», amounts to 560 % in advanced models of reverse heat pumps of the air-water type [7].

The values of HP and RM efficiency are greater than unity, which seems to contradict the general efficiency definition, but this contradiction can be explained from the point of view of modern natural science. The initial classic definition of efficiency had respect to conversion of the same kind of energy – mechanical work. In thermodynamics this indicator started to be used as a characteristic of heat conversion into work, thus departing from its classical understanding. The second peculiarity of this explanation is the restriction of application of the second law of thermodynamics only to equilibrium phenomena devoid of self-organization [8]. In HP and RM the self-organizing phase transfer – evaporation-condensation – is used. In its nature This self-organizing process is energy-saving in its nature and for this reason is not in keeping with the second law of thermodynamics.

In their monograph «The Exergy Method and Its Application» V.M. Brodyansky with co-authors paid great attention to the problem of using the indicator of efficiency in relation to HP and RM, as well as to the fact that it is greater than unity. They point out [4, p. 137], that: «Academician A.A. Kharkevich showed overwhelmingly deeper understanding of difficulties in scientific determination of efficiency. He wrote: «Methodological difficulties here are of fundamental character. The major of them … is to specify what should be considered as useful effect of this device, and to find the quantitative measure of this useful effect». The author considers this problem as «nothing but insurmountable under the current understanding of the essence of the matter».

Under «the current understanding» A.A. Kharkevich meant [4, p. 137]: «… the use as… the values of energy of various types, chosen from the energy balance, that is, eventually the discrepancy between the notion of useful efficiency coefficient (UEC) and requirement to efficiency». This approach has given birth to pessimistic view that «elegant simplicity of the initial definition of efficiency has been irrevocably lost». Actually, the initial definition of efficiency (effect utile) introduced (late XVIII – early XIX centuries) by French scientists Carnot, Navier, Poncelet, and developed by Coriolis, was elegantly simple: efficiency is the ratio of useful work to energy expended. As it were mechanical and hydraulic machines that were assessed, such definition was strict enough and met all the requirements, that is, efficiency was always lesser than unity and difference between the numerator and the denominator and was exactly equal to the loss. Consequently, the efficiency definition was strictly unequivocal – no other values cant be obtained from the relation of these two kinds of work. This is explained by the fact that compared values were qualitatively similar – work at the input and work at the output. However, as soon as this approach was extended to the «realm of thermal engineering» – namely, to heat engines wherein qualitatively different kinds of energy were compared in transition «heat-work», «elegant simplicity» was really lost, as the single efficiency was transformed into various transformation coefficients not meeting the requirements to the classic efficiency definition.

## Conclusion

The substantiation of the exergy analysis of ambient heat conversion – the most available energy source – as well as the understanding of relevance of the use of values of thermodynamic and exergy efficiency in relation to HP and RM with the same degree of reliability as to heat engines, remove all doubts in high energy efficiency of these energy converters. Rapid modernization of energy supply solutions for the agroindustrial complex and the housing and utility sector is urgent: it is necessary to replace worn and outdated fuel heat generators by mini thermoelectric plants with cogeneration and the use of HP.

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# ЛОГИКО-МАТЕМАТИЧЕСКИЙ И ЭКСЕРГЕТИЧЕСКИЙ АНАЛИЗЫ В ОБОСНОВАНИИ КПД ТЕПЛОВЫХ НАСОСОВ И ХОЛОДИЛЬНИКОВ

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Выявлена возможность и разработана методика эксергетического анализа преобразований теплоты среды используемой тепловыми насосами (ТН) и холодильными машинами (ХМ) – наиболее эффективными генераторами теплоты и холода. Логико-математическим анализом Обоснована целесообразность использования применительно к ТН и ХМ показателей термодинамический и эксергетический КПД с такой же правомочностью, как и к тепловым силовым машинам. Отсутствие обоснованности применения показателей термодинамический и эксергетический КПД к ТН вызывало недоверие к их высокой эффективности преобразования электроэнергии в низкотемпературную тепловую энергию. Это привело к отставанию РФ в производстве и применении ТН. Из общего количества ежегодно потребляемого в РФ топлива более 50 % затрачивают только для получения низкотемпературного тепла. Эксэргия этой части топлива не используют. Это привело к высокой энергоемкости сельскохозяйственной продукции и внутреннего валового продукта РФ.

Необходима неотложная модернизация энергообеспечения АПК и ЖКХ РФ: замена традиционных котельных на мини-ТЭЦ с когенерацией и использованием ТН.

**Ключевые слова:** холодильник, тепловой насос, эксергетический анализ, термодинамический КПД, эксергетический КПД, энергоёмкость, мини-ТЭЦ, когенерация.

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# COMPUTING THE ROOTS OF CYCLOTOMIC EQUATIONS IN LIANIT ALGEBRAS

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*We apply the formalism of lianit theory to calculate the n<sup>th</sup> roots of cyclotomic equations.*
***Keywords**: cyclotomic equations, $n^{th}$ roots of unity, lianit roots, transforming polynomials, dummy index identities*

## Introduction

In paper [1] the general-theoretical groundwork for solving cyclotomic equations (CE) [2], [3], [4] of any degree $f^n(x) = x^n - 1$ were laid. A universal formula for the elements of the lianit $f^n(\sigma)$ was obtained and the existence of $p, q$-parameter polynomial $\varphi_1^{n-1}(p, q)$ was proved (here $\sigma = \left(-p, \frac{q}{p}\right)$, see the Theorem and the formula (2) thereto in [1]). In [1] it was shown that the problem of solving $n^{th}$ degree CE reduces to construing a certain polynomial of degree $\ell = \frac{n-1}{2}$. For these polynomials the term *transforming polynomial* (TP) was introduced. The roots of TP were shown to satisfy the so called *dummy index identities* $p_i p_j = -(p_v + p_w)$ $(i \neq j,\ v \neq w)$. An efficient, universal algorithm of finding TP for any CE was also provided in [1]. In this paper we compute the roots of cyclotomic (and related elementary palindromic) equations for $n$ up to 32 based on the lianit formalism developed in papers [5], [6], [7], [8], [9].

The paper is organized as follows. In Section 2 we calculate in detail the $n^{th}$ roots of unity for n=13, 17, 19. We conclude this section by stating the perfectly analogous cases of n=23, 29, 31, considering by way of illustration, n=31. In Section 3, we provide a universal algorithm of solving any CE of even degree $n$ regardless of its palindromic or antipalindromic nature i.e. the cases $f^n(x) = x^n \pm 1$. This discussion exhausts all possible cases of CE up to $32^{nd}$ degree. The results of our work are discussed in the Summary.

## Finding the $n^{th}$ Roots of Unity

As in [1] all computations are carried out within the algebra,

$$\sigma_1 \cdot \sigma_2 = (x_1, x_2)(y_1, y_2) = [x_1(y_1 + y_2), x_2 y_1] \quad (1)$$

with a right-hand unity $e = (1, 0)$ and $k = (k, 0)$ as the analogue of the complex number $k$.

To calculate the function $\varphi_1^{n-1}(p, q)$ explicitly one simply performs element by element identity comparison between $f^n(\sigma) = \sigma^n + \sigma^{n-1} \cdot a_1 + \cdots + \sigma \cdot a_{n-1} + a_n$ and the universal expression in (2) from [1]. The primitive roots for cases $n = 5, 7, 11$ were calculated earlier in papers [7], [8] using the theorem of principal lianit roots (see [1], [2] or [9]). In [1] the case of $n = 11$ was solved by way of illustrating the developed theory. In case of $x^{13} - 1 = 0$, based on (13)-(14) from [1], TP reads,

$$\begin{aligned} f^6(p) &= p^6 - p^5 - 5p^4 + 4p^3 + 6p^2 - 3p - 1 = \\ &= (p^2 + a_1 p + a_2)(p^4 + b_1 p^3 + b_2 p^2 + b_3 p + b_4) \end{aligned} \quad (2)$$

the required coefficients $a_1, a_2$, are evidently the complex roots of the one and the same equation of n=15 degree, deriving from the index identity for $f^6(p)$ as we have $\frac{6!}{2! \cdot 4!} = 15$. In contrast to the case of $x^{11} - 1 = 0$, the polynomials $f^{15}(a_1) \equiv f^{15}(a_2)$ permit cubic integral factors. Indeed, out of six roots $p_1, p_2, \dots, p_6$ of TP $f^6(p)$ one distinguishes the following pairs guided by the dummy indices,

$$for\ a_1 \to \varepsilon_1 = -(p_1 + p_2);\ \ \varepsilon_2 = -(p_3 + p_4);\ \ \varepsilon_3 = -(p_5 + p_6);$$

$$for\ \ a_2 \to \delta_1 = p_1 p_3, \delta_2 = p_2 p_4, \delta_3 = p_5 p_6.$$

Denote the required factors by: $f^3(a_1) \equiv f^3(a_2) = a_1^3 + k_1a_1^2 + k_2a_1 + k_3 = a_2^3 + k_1a_2^2 + k_2a_2 + k_3$. Then one has: $-k_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = -(p_1 + p_2 + p_3 + p_4 + p_5 + p_6) = -1$. The coefficient $k_3$ also equals to unity, since $k_3 = -\delta_1\delta_2\delta_3 = -p_1p_2p_3p_4p_5p_6 = -(-1) = 1$. For $k_2$ we have: $k_2 = \varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_3 = (p_1 + p_2)(p_3 + p_4) + (p_1 + p_2)(p_5 + p_6) + (p_3 + p_4)(p_5 + p_6)$. In all 12 summands $p_ip_j$ $(i \neq j)$, all the dummy indices appear symmetrically. In view of the identity $p_ip_j = -(p_v + p_w)$, from 12 summands $p_ip_j$ we obtain exactly 24 free terms $p_\ell$ with sign $(-)$. In other words: $k_2 = -4(p_1 + p_2 + p_3 + p_4 + p_5 + p_6) = -4$. For the sought for polynomials $f^3(a_1)$, $f^3(a_2)$ we can now write: $f^3(a_1) = a_1^3 + a_1^2 - 4a_1 + 1$; $f^3(a_2) = a_2^3 + a_2^2 - 4a_2 + 1$. In expansion (2) it is self-evident that when selecting a specific value of $a_1$, the value of the coefficient $a_2 \neq a_1$. It is noteworthy, that the roots of the TP $f^6(p) = p^6 - p^5 - 5p^4 + 4p^3 + 6p^2 - 3p - 1$ may be also found through a different factorization scheme,

$$f^6(p) = p^6 - p^5 - 5p^4 + 4p^3 + 6p^2 - 3p - 1 \\ = (p^3 + a_1p^2 + a_2p + a_3)(p^3 + b_1p^2 + b_2p + b_3). \quad (3)$$

In (3) $a_1$ and $b_1$ are some possible combinations of triplets, $\varepsilon_\ell = -(p_i + p_j + p_k)$ $(i \neq j \neq k)$. There are exactly $N = \frac{6!}{3!\cdot 3!} = 20$ such triplets out of six values of $p_i$ $(i = 1, 2, \dots, 6)$. Among these, in particular, exist the pairs $\varepsilon_1 = -(p_1 + p_2 + p_3)$ and $\varepsilon_2 = -(p_3 + p_4 + p_5)$, which, in accordance with the index identity relations, must guarantee the existence of an integral trinomial $f^2(a_1) = a_1^2 + k_1a_1 + k_2$. Hence we have: $-k_1 = \varepsilon_1 + \varepsilon_2 = -(p_1 + p_2 + \dots + p_6) = -1$, i.e. $k_1 = 1$. At the same time: $k_2 = \varepsilon_1\varepsilon_2 = (p_1 + p_2 + p_3)(p_4 + p_5 + p_6)$. The nine summands $p_ip_j$ $(i \neq j)$, lead to exactly 18 free terms $p_\ell$ with the sign $(-)$. Hence: $k_2 = -3(p_1 + p_2 + \dots + p_6) = -3$. Therefore, the equation $f^{20}(a_1) = 0$, the numeric roots thereof are the triplets $\varepsilon_\ell = -(p_i + p_j + p_k)$, has a factor polynomial $f^2(a_1) = a_1^2 + k_1a_1 + k_2 = a_1^2 + a_1 - 3$, hence in (3) $a_1 = \frac{-1+\sqrt{13}}{2}$, $b_1 = \frac{-1-\sqrt{13}}{2}$. The possible values of $a_2$ и $b_2$, are easily found via analogous reasinong i.e. choosing dummy indices: $\delta_1 = p_1p_2 + p_1p_3 + p_2p_3$ и $\delta_2 = p_4p_5 + p_4p_6 + p_5p_6$. The coefficients $a_2, b_2$ are trivially shown to be roots of $f^2(a_2) = a_2^2 + 2a_2 + 1$, т.е. $a_2 = b_2 = -1$. Subsequently

$$f^6(p) = \left[p^3 + \frac{-1+\sqrt{13}}{2}p^2 - p + \frac{3-\sqrt{13}}{2}\right]\left[p^3 + \frac{-1-\sqrt{13}}{2}p^2 - p + \frac{3+\sqrt{13}}{2}\right]. \quad (4)$$

The six values of $p_i$ yield six quadratic equations $x^2 + p_ix + 1 = 0$, whose numeric roots are the primitive roots of $x^{13} - 1 = 0$, implying that the lianit roots $\sigma_i = \left(-p_i, \frac{1}{p_i}\right)$ are secondary lianit roots to $f^{13}(x) = x^{13} - 1$ (the total number of such secondary roots $N = \frac{13!}{2!\cdot 11!} = 78$).

3. The TP for the much-famed case of Gauss $f^{17}(x) = x^{17} - 1$, is given by,

$$f^8(p) = p^8 - p^7 - 7p^6 + 6p^5 + 15p^4 - 10p^3 - 10p^2 + 4p + 1. \quad (5)$$

Denote $f^2(p) = p^2 + a_1p + a_2$ as possible factor for $f^8(p)$, where $a_1, a_2$ are the numeric roots of one and the same equations of degree $N = \frac{8!}{2!\cdot 6!} = 28$. Because of the index identities $p_ip_j = -(p_v + p_w)$ the polynomials $f^{28}(a_1) \equiv f^{28}(a_2)$ permit integral factors,

$$f^4(a_1) \equiv f^4(a_2) = a_1^4 + k_1a_1^3 + k_2a_1^2 + k_3a_1 + k_4 = a_2^4 + k_1a_2^3 + k_2a_2^2 + k_3a_2 + k_4. \quad (6)$$

Considering the dummy index pairs,

$$a_1 \to \ \varepsilon_1 = -(p_1 + p_2);\ \ \varepsilon_2 = -(p_3 + p_4);\ \ \varepsilon_3 = -(p_5 + p_6);\ \ \varepsilon_4 = -(p_7 + p_8), \\ a_2 \to \ \delta_1 = p_2p_4,\quad \delta_2 = p_1p_5,\quad \delta_3 = p_7p_8,\quad \delta_4 = p_3p_6. \quad (7)$$

for the coefficients (6) we have: $-k_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 = -(p_1 + p_2 + \dots + p_8) = -1$. (obviously $k_4 = \delta_1\delta_2\delta_3\delta_4 = p_1 \cdot p_2 \cdots p_8 = 1$ (obviously $k_4$ can be found without turning to the second set of (7), writing simply $k_4 = \varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4$). In the equation for $k_2 = \varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_1\varepsilon_4 + \varepsilon_2\varepsilon_3 + \varepsilon_2\varepsilon_4 + \varepsilon_3\varepsilon_4$ enter exactly 24 summands $p_ip_j$ $(i \neq j)$, each of which, in view of the index identities, gives two free terms $p_e$ with the $(-)$sign. These 48 symetrically occuring $(-p_\ell)$ give: $k_2 = -6(p_1 + p_2 + \dots + p_8) = -6$. Analogously, $-k_3 = \varepsilon_1\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_2\varepsilon_4 + \varepsilon_1\varepsilon_3\varepsilon_4 + \varepsilon_2\varepsilon_3\varepsilon_4$. Following the index identity $p_ip_j = -(p_v + p_w)$ we know that the structure of each summand is identical to the others.. Consider, for instance, $\varepsilon_1\varepsilon_2\varepsilon_3 = -(p_1 + p_2)(p_3 +$

$p_4)(p_5+p_6)$. The expression $-(p_1+p_2)(p_3+p_4)$ gives 4 summands $-p_ip_j$, equivalent to eight free terms $+p_\ell$. When multiplying these eight terms $p_\ell$ by $(p_5+p_6)$, by we receive exactly 16 products $p_ip_j$. Among these products there are pairs with coinciding indices. From symmetry considerations, each of the four summands $\varepsilon_i\varepsilon_j\varepsilon_k$ must involve two products $p_i\cdot p_i$ such as to make up a symmetric function $p_1^2+p_2^2+\cdots+p_8^2$. Therefore the product $\varepsilon_1\varepsilon_2\varepsilon_3$ involves exactly 14 products $p_ip_j$ $(i\neq j)$ and two pairs with coinciding indices i.e. exactly 28 free terms $(-p_\ell)$ and the sum $p_i^2+p_j^2$ $(i\neq j)$. These reasonings lead to: $-k_3=\varepsilon_1\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_2\varepsilon_4+\varepsilon_1\varepsilon_3\varepsilon_4+\varepsilon_2\varepsilon_3\varepsilon_4=(p_1^2+p_2^2+\cdots+p_8^2)-\frac{4\cdot 28}{8}\cdot(p_1+p_2+\cdots+p_8)=15-14=1$, or $k_3=-1$. Therefore,

$$f^4(a_1)\equiv f^4(a_2)=a_1^4+a_1^3-6a_1^2-a_1+1=a_2^4+a_2^3-6a_2^2-a_2+1. \tag{8}$$

which in its turn permits the factorization,

$$f^4(a_1)=\left[a_1^2+\frac{1+\sqrt{17}}{2}a_1-1\right]\left[a_1^2+\frac{1-\sqrt{17}}{2}a_1-1\right]. \tag{9}$$

Therefore TP (5) expands as,

$$\begin{aligned}f^8(p)=&\left[p^2+\frac{-1-\sqrt{17}-\sqrt{2(17+\sqrt{17})}}{4}p+\frac{-1+\sqrt{17}+\sqrt{2(17-\sqrt{17})}}{4}\right]\times\\ &\times\left[p^2+\frac{-1-\sqrt{17}+\sqrt{2(17+\sqrt{17})}}{4}p+\frac{-1+\sqrt{17}-\sqrt{2(17-\sqrt{17})}}{4}\right]\times\\ &\times\left[p^2+\frac{-1+\sqrt{17}-\sqrt{2(17-\sqrt{17})}}{4}p+\frac{-1-\sqrt{17}-\sqrt{2(17+\sqrt{17})}}{4}\right]\times\\ &\times\left[p^2+\frac{-1+\sqrt{17}+\sqrt{2(17-\sqrt{17})}}{4}p+\frac{-1-\sqrt{17}+\sqrt{2(17+\sqrt{17})}}{4}\right].\end{aligned} \tag{10}$$

Denote the roots of the quadratic trinomial in (10) as $(p_1,p_2)$; $(p_3,p_4)$; $(p_5,p_6)$; $(p_7,p_8)$, then elementary calculation shows, $p_1p_2=-(p_7+p_8)$; $p_3p_4=(-p_5+p_6)$; $p_5p_6=-(p_1+p_2)$; $p_7p_8=-(p_3+p_4)$, in full accordance with (7). Thus, the numeric roots of each eight quadratic trinomials $f^2(x)=x^2+p_ix+1$ form a set of 16 primitive roots of the equation $x^{17}-1=0$, evidently expressible in square radicals (as $n=17$ is a Fermat number). We observe, that TP $f^8(p)$ also permits the expansion,

$$f^8(p)=(p^4+a_1p^3+a_2p^2+a_3p+a_4)(p^4+b_1p^3+b_2p^2+b_3p+b_4). \tag{11}$$

This is the decomposition that yields the well-known chain of resolving equations related to Gaussian periods. The coefficients $a_1$ and $b_1$, assume the values of all possible sums $\varepsilon_i=-(p_{i_1}+p_{i_2}+p_{i_3}+p_{i_4})$, which count to $N=70$. Among these sums there is surely the pair $\varepsilon_1=-(p_1+p_2+p_3+p_4)$; $\varepsilon_2=-(p_5+p_6+p_7+p_8)$, which enruses the existence of an integral factor polynomial $f^2(a_1)=a_1^2+k_1a_1+k_2$ for $f^{70}(a_1)$. We have indeed: $-k_1=\varepsilon_1+\varepsilon_2=-(p_1+p_2+\cdots+p_8)=-1$, i.e. $k_1=1$; $k_2=\varepsilon_1\cdot\varepsilon_2=(p_1+p_2+p_3+p_4)(p_5+p_6+p_7+p_8)$. The coefficient $k_2$ involves exactly 16 factors $p_ip_j$ $(i\neq j)$. The index identity relations $p_ip_j=-(p_v+p_w)$ render these factors into 32 free terms $(-p_\ell)$, which implies $k_2=-4(p_1+p_2+\cdots+p_7+p_8)=-4$. As stated above, we have received the first chain of resolving Gaussian equations, namely: $a_1^2+a_1-4=0$, hence: $a_1,b_1=\frac{-1\pm\sqrt{17}}{2}$. Although the knowledge of the values of $a_1,b_1$ is quite sufficient for the calculation of the rest of the coefficients of the polynomials (11), it is still instructive to perform their computation via the dummy index identities. Evidently $a_2$ and $b_2$ can be written as,

$$\delta_1 = p_1p_2 + p_1p_3 + p_1p_4 + p_2p_3 + p_2p_4 + p_3p_4;$$
$$\delta_2 = p_5p_6 + p_5p_7 + p_5p_8 + p_6p_7 + p_6p_8 + p_7p_8. \quad (12)$$

which simply states that $\delta_1$ and $\delta_2$ are roots to $f^2(a_2) = a_2^2 + k_1a_2 + k_2$ with integral coefficients $k_1, k_2$. We have: $-k_1 = \delta_1 + \delta_2 = -3(p_1 + p_2 + \cdots + p_8) = -3$, since 12 summands $p_ip_j$ $(i \neq j)$ give 24 free terms $(-p_e)$, hence $k_1 = 3$. From symmetry considerations it is clear that $\delta_1$ and $\delta_2$ must have identical structures and since each involves exactly 12 free terms $(-p_\ell)$, it follows: $\delta_1 = [-2(p_1 + p_2 + p_3 + p_4) - (p_5 + p_6 + p_7 + p_8)]$; $\delta_2 = [-2(p_5 + p_6 + p_7 + p_8) - (p_1 + p_2 + p_3 + p_4)]$. Subsequently: $k_2 = \delta_1\delta_2 = 2(p_1^2 + p_2^2 + \cdots + p_7^2 + p_8^2) - \frac{256}{8}(p_1 + p_2 + \cdots + p_7 + p_8) = 2 \cdot 15 - 32 = -2$. Thus, $f^2(a_2) = a_2^2 + 3a_2 - 2$ and choosing $a_1, b_1 = \frac{-1\pm\sqrt{17}}{2}$, for $a_2$ and $b_2$ we obtain: $a_2, b_2 = \frac{-3\mp\sqrt{17}}{2}$. Analogously can be shown, that $a_3, b_3 = -2 \mp \sqrt{17}$; $a_4, b_4 = -1$. Therefore,

$$f^8(p) = \left[p^4 + \frac{-1+\sqrt{17}}{2}p^3 + \frac{-3-\sqrt{17}}{2}p^2 + \left(-2-\sqrt{17}\right)p - 1\right]\left[p^4 + \frac{-1-\sqrt{17}}{2}p^3 + \right. \\ \left. + \frac{-3+\sqrt{17}}{2}p^2 + \left(-2+\sqrt{17}\right)p - 1\right]. \quad (13)$$

Each of the brackets in (13) in its own turn expands into quadratic trinomials with coefficients expressible in square radicals. For example, take the first bracket,

$$p^4 + \frac{-1+\sqrt{17}}{2}p^3 + \frac{-3-\sqrt{17}}{2}p^2 + \left(-2-\sqrt{17}\right)p - 1 == (p^2 + \eta_1p + \eta_2)(p^2 + \eta_3p + \eta_4) \quad (14)$$

where $\eta_1, \eta_3$ are the roots of $z^2 + \frac{1-\sqrt{17}}{2}z - 1 = 0$ and $\eta_2, \eta_4$ are the roots of $z^2 + \frac{1+\sqrt{17}}{2}z - 1 = 0$ (the equations thereof correspond to the quartic Gaussian periods). We have,

$$\eta_3, \eta_1 = \frac{\left(-1+\sqrt{17}\right) \pm \sqrt{2\left(17-\sqrt{17}\right)}}{4}\ ; \ \eta_4, \eta_2 = \frac{\left(-1-\sqrt{17}\right) \pm \sqrt{2\left(17+\sqrt{17}\right)}}{4}. \quad (15)$$

4. For the equation $x^{19} - 1 = 0$ the TP is given by,

$$f^9(p) = p^9 - p^8 - 8p^7 + 7p^6 + 21p^5 - 15p^4 - 20p^3 + 10p^2 + 5p - 1. \quad (16)$$

The most effecient decomposition scheme for (16) is,

$$f^9(p) = (p^3 + a_1p^2 + a_2p + a_3)(p^3 + b_1p^2 + b_2p + b_3)(p^3 + c_1p^2 + c_2p + c_3). \quad (17)$$

The values of coefficients $a_1, b_1, c_1$ appear as all possible sums $-\left(p_{i_1} + p_{i_2} + p_{i_3}\right)$, i.e. the equation which numeric roots are equal to thereof sums has a degree of $N = \frac{9!}{3!\cdot 6!} = 84$. We indicate triplets of roots $p_i$ $(i = 1, 2, \dots, 9)$ permitting the equation thereof to have factors in the form, $f^3(a_1) = a_1^3 + k_1a_1^2 + k_2a_1 + k_3$, where $k_1, k_2, k_3$ are integers. Indeed, if one considers the triplets,

$$a_1 \rightarrow \varepsilon_1 = -(p_1 + p_2 + p_3); \ \varepsilon_2 = -(p_4 + p_5 + p_6); \ \varepsilon_3 = -(p_7 + p_8 + p_9). \quad (18)$$

Then, $-k_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = -[p_1 + p_2 + \cdots + p_8 + p_9] = -1$, i.e. $k_1 = 1$. Next, $k_2 = \varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_3$, and since each of these three summands contains 9 products $p_ip_j$ $(i \neq j)$, the coefficient $k_2$ is made of 27 products $p_ip_j$. The dummy index identities then give us exactly 54 free terms $(-p_\ell)$, thus we compute $k_2 = \varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_3 = -6(p_1 + p_2 + \cdots + p_8 + p_9) = -6$. Analogously, $k_3 = -\varepsilon_1\varepsilon_2\varepsilon_3 = (p_1 + p_2 + p_3)(p_4 + p_5 + p_6)(p_7 + p_8 + p_9)$. Obviously, the product $(p_1 + p_2 + p_3)(p_4 + p_5 + p_6)$ gives 9 products $p_ip_j$ $(i \neq j)$, and therefore 18 free terms $(-p_e)$ with the sign $(-)$. Their multiplication with $(p_7 + p_8 + p_9)$ gives exactly 54 products $p_ip_j$, out of which 9 are products $p_ip_j$ with coinciding indices, i.e. $-(p_1^2 + p_2^2 + \cdots + p_8^2 + p_9^2) = -(1 + 16) == -17$. The rest of the 45 products $p_ip_j$ $(i \neq j)$ with the sign $(-)$ give exactly 90 free terms $p_\ell$ with the sign $(+)$, i.e. $10(p_1 + p_2 + \cdots + p_8 + p_9) = +10$. Combining these results, we receive $k_3 = -17 + 10 = -7$. Thus, the polynomial $f^{84}(a_1)$ has a factor in the form of $f^3(a_1) = a_1^3 + a_1^2 - 6a_1 - 7$. In (17) the coefficients $a_1, b_1, c_1$ are essentially the roots of $f^3(a_1) = a_1^3 + a_1^2 - 6a_1 - 7$. The values of the rest of of the coefficients $(a_2, b_2, c_2)$; $(a_3, b_3, c_3)$ can be found with analogous reasoning. The triplet $(a_2, b_2, c_2)$ corresponds to the combination $p_1p_2 + p_1p_3 + p_2p_3$; $p_4p_5 + p_4p_6 + p_5p_6$; $p_7p_8 + p_7p_9 + p_8p_9$ (these, obviously, are dummy indices). They are the complex roots of $f^3(a_2) = a_2^3 + 2a_2^2 - 5a_2 + 1$. The triplet $(a_3, b_3, c_3)$ is root to $f^3(a_3) = a_3^3 - 5a_3^2 + 2a_3 + 1$. The triplet thereof corresponds to $(a_3, b_3, c_3) = (-p_1p_2p_3, -p_4p_5p_6, -p_7p_8p_9)$. With the help of substitution $a_1 = z - \frac{1}{3}$, the polynomial $f^3(a_1)$ is modified to $f^3(z) = z^3 - \frac{19}{3}z - \frac{7\cdot 19}{27}$. We have,

$$a_1 = -\frac{1}{3} + \sqrt[3]{\frac{7}{18}\left(\frac{19}{3} + i\sqrt{3}\right)} + \sqrt[3]{\frac{7}{18}\left(\frac{19}{3} - i\sqrt{3}\right)} = x + y - \frac{1}{3}. \tag{19}$$

Ultimately, for $f^9(p) = p^9 - p^8 - 8p^7 + 7p^6 + 21p^5 - 15p^4 - 20p^3 + 10p^2 + 5p - 1 == (p^3 + a_1p^2 + a_2p + a_3)(p^3 + b_1p^2 + b_2p + b_3)(p^3 + c_1p^2 + c_2p + c_3)$ we have: $a_1 = x + +y - \frac{1}{3}$; $b_1 = Bx + Dy - \frac{1}{3}$; $c_1 = Dx + By - \frac{1}{3}$; $a_2 = -Bx - Dy - \frac{2}{3}$; $b_2 = -Dx - By - \frac{2}{3}$; $c_2 = -x - y - \frac{2}{3}$; $a_3 = x + y + \frac{5}{3}$; $b_3 = Bx + Dy + \frac{5}{3}$; $c_3 = Dx + By + \frac{5}{3}$, where $B, D = \frac{-1 \pm i\sqrt{3}}{2}$. The next cases $n = 23, 29, 31$ are solved in complete analogy with the case of $n = 11$ involving Lagrange's resolvents [1]. By way of illustration, consider $f^{31}(x) = x^{31} - 1$. TP calculates as,

$$f^{15}(p) = p^{15} - p^{14} - 14p^{13} + 13p^{12} + 78p^{11} - 66p^{10} - 220p^9 + 165p^8 + 330p^7 - 210p^6 - 252p^5 + 126p^4 + 84p^3 - 28p^2 - 8p + 1. \tag{20}$$

If we expand $f^{15}(p)$ as $f^{15}(p) = (p^5 + a_1p^4 + \cdots + a_5)(p^5 + b_1p^4 + \cdots + b_5)(p^5 + c_1p^4 + \cdots + c_5)$, the coefficients $(a_i, b_i, c_i)$ are readily calculated via the dummy index identities. For example, the triplet $(a_1, b_1, c_1)$ are the roots of $f^3(a_1) = a_1^3 + a_1^2 - -10a_1 + 23$. However, to compute the numeric roots of all three quintic polynomials from (20), the joint application of Lagrange's resolvents and the dummy index identities is necessary.

**Elementary palindromic and antipalindromic equations**

The algorithm for constructing TPs for any $f^n(x) = x^n - 1$, where $n$ is an even number, was formulated in paper [1]. Using the algorithm (34)-(35) from [1], we have,

$$\begin{aligned}
&x^6 - 1 = 0; \quad f^3(p) = p^3 - 2p^2 - p + 2,\\
&x^8 - 1 = 0; \quad f^4(p) = p^4 - 2p^3 - 2p^2 + 4p,\\
&x^{10} - 1 = 0; \quad f^5(p) = p^5 - 2p^4 - 3p^3 + 6p^2 + p - 2,\\
&x^{12} - 1 = 0; \quad f^6(p) = p^6 - 2p^5 - 4p^4 + 8p^3 + 3p^2 - 6p,\\
&x^{14} - 1 = 0; \quad f^7(p) = p^7 - 2p^6 - 5p^5 + 10p^4 + 6p^3 - 12p^2 - p + 2,\\
&x^{16} - 1 = 0; \quad f^8(p) = p^8 - 2p^7 - 6p^6 + 12p^5 + 10p^4 - 20p^3 - 4p^2 + 8p.
\end{aligned} \tag{21}$$

The secondary lianit roots of $x^n - 1 = 0$, within the set (1), at any even $n$, are given by $\sigma_i = \left(-p_i, \frac{1}{p_i}\right)$, where $p_i$ are the numeric roots of $f^{\frac{n}{2}}(p)$. The numeric roots of the equation $x^n - 1 = 0$ are the roots of the quadratic equations $x^2 + p_ix + 1 = 0$; $(i = 1, 2, \ldots, \frac{n}{2})$. It is peculiar, that the numeric roots $p_i$ of TPs from (21) do not satisfy the dummy index identities. Only in select special cases when $\frac{n}{2}$ is an odd number (at even $n$), there appear factor polynomials with such property (This is the case, for example, in $x^{10} - 1 = 0$, $x^{14} - 1 = 0$). Indeed, $f^5(p) = (p - 2)(p^2 - p - 1)(p^2 + p - 1)$, where the trinomials

$f^2(p) = p^2 \mp p - 1$ correspond to the TPs of equations $x^5 \mp 1 = 0$. The problem of calcuating the roots of $x^n + 1$, for $n = 8, 16, 32, \ldots$ is not trivial. Following [1], some cases are brought out below,

$$\begin{aligned} &x^6 + 1 = 0; \quad f^3(p) = p^3 - 3p, \\ &x^8 + 1 = 0; \quad f^4(p) = p^4 - 4p^2 + 2, \\ &x^{10} + 1 = 0; \quad f^5(p) = p^5 - 5p^3 + 5p, \\ &x^{12} + 1 = 0; \quad f^6(p) = p^6 - 6p^4 + 9p^2 - 2, \\ &x^{14} + 1 = 0; \quad f^7(p) = p^7 - 7p^5 + 14p^3 - 7p, \\ &x^{16} + 1 = 0; \quad f^8(p) = p^8 - 8p^6 + 20p^4 - 16p^2 + 2. \end{aligned} \tag{22}$$

The methodological and computational advantages of solving quadratic equations $x^2 + p_i x + 1 = 0$ $(i = 1, 2, \ldots, l = \frac{n}{2})$ via the roots $p_i$ of corresponding TPs over the conventional method of computing the roots of $x^n - 1 = 0$ and their subsequent multiplication by one of various values of $\sqrt[n]{-1}$ are obvious. By way of illustration, solve the equation $x^{32} + 1 = 0$. The TP reads,

$$f^{16}(p) = p^{16} - 16p^{14} + 104p^{12} - 352p^{10} + 660p^8 - 672p^6 + 346p^4 - 64p^2 + 2. \tag{23}$$

Denoting $z = p^2$ we rewrite (23) as,

$$\begin{aligned} f^8(z) &= \left(z^4 - 8z^3 + 20z^2 - 16z + 2 + \sqrt{2}\right)\left(z^4 - 8z^3 + 20z^2 - 16z + 2 - \sqrt{2}\right) = \\ &= \left(z^2 - 4z + 2 + \sqrt{2 - \sqrt{2}}\right)\left(z^2 - 4z + 2 - \sqrt{2 - \sqrt{2}}\right)\left(z^2 - 4z + 2 + \sqrt{2 + \sqrt{2}}\right) \cdot \\ &\cdot \left(z^2 - 4z + 2 - \sqrt{2 + \sqrt{2}}\right). \end{aligned}$$

Within the set (1) the equation $x^{32} + 1 = 0$ holds $N = 496$ lianit roots ($N = \frac{32!}{2!30!} = 496$). Using the TP (22), the sixteen of these lianit solutions $\sigma_i = \left(p_i, \frac{1}{p_i}\right)$ generate quadratic equations $x^2 + p_i x + 1 = 0$, with numeric roots coinciding with those of $x^{32} + 1 = 0$. For the first bracket of (23), for instance, we have,

$$p_{1,2} = \pm\sqrt{2 + \sqrt{2 - \sqrt{2 - \sqrt{2}}}}; \quad p_{3,4} = \pm\sqrt{2 - \sqrt{2 - \sqrt{2 - \sqrt{2}}}}. \tag{24}$$

**Summary**

To conclude we summarize the results of our work. We have computed the n^th roots of cyclotomic equations $x^n \pm 1 = 0$ up to n=32 based on the lianit-algebraic theory of cyclotomic equations developed in our earlier works. The obtained results perspicuously demonstrate how the lianit algorithm renders the problem of cyclotomic equations into a standard computational procedure. At the same time, each stage of computation has a unified algebraic interpretation containing all the known results of Galois Group Theory. The provided calculation of cyclotomic equations shows that while the Standard Group Theory is essential for modern Algebra and applications, it is not necessarily well-suited for solving algebraic equations, especially at higher degrees.

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## ВЫЧИСЛЕНИЕ ПЕРВООБРАЗНЫХ КОРНЕЙ ИЗ ЕДИНИЦЫ В ЛИАНИТОВЫХ АЛГЕБРАХ

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В статье представлен расчет первообразных корней из единицы (ПКЕ) для всех степеней $n$ от 5 до 32 на основании формализма лианитовой теории уравнений деления круга (УДК). Особое внимание уделено случаям n=13, 17, 19 представляющим значительный научный интерес, в том числе и для приложений. Проводится подробная схема расчетов ПКЕ для n=23, 29, 31. Рассмотрены также корни так называемых *возвратных* уравнений для случаев симметрических и антисимметрических коэффициентов. Обобщается схема составления переходных многочленов (ПМ), введенных в предыдущем статье для этих уравнений, указывается практический алгоритм для расчета их числовых корней. Для всех степеней (четных и нечетных) $n$ от 5 до 32 представлены алгоритмы расчета коэффициентов ПМ. Выписаны в явном виде так называемые *тождества условных индексов*, которым удовлетворяют корни ПМ. Подробно обсуждается схема факторизации ПМ на элементарные полиномиальные сомножители. Доказывается, что коэффициенты разложения ПМ являются однозначными симметрическими функциями от условных индексов, устанавливается знакопеременность этих функций. При рассмотрении знаменитого случая правильного семнадцатиугольника, как частный случай из общей лианитовой теории УДК, следует цепочка разрешающих уравнений для Гауссовских периодов. Этот и другие примеры УДК и возвратных уравнений наглядно демонстрируют превосходство лианитовых методов решения алгебраических уравнений над стандартными теоретико-групповыми методами теории Галуа.

**Ключевые слова: у**равнения деления круга, первообразные корни из единицы, лианитовые корни, переходной многочлен, тождества условных индексов, Гауссовские периоды.

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