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INVESTIGATION OF THE MIRROR DEFLECTING OPTICAL SYSTEMS FOR SOLAR CONCENTRATOR MODULES

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The results of investigation of deflecting optical system providing 100% passing of reflected beams at the exit surface are presented. Optical calculation of ray path is given and the concentration factor of solar modules with a mirror deflecting optical system and various types of concentrators is considered. One of the trends of reduction of photovoltaic equipment cost is the use of photovoltaic modules with solar concentrators. The purpose of this work is the development of the concentrator solar module with low optical losses and high uniformity of illumination at a radiation receiver, as well as with high specific capacity of the radiation receiver. One of simple and effective methods of providing uniform distribution of concentrated solar radiation at the receiver surface is the use of mirror deflecting optical systems (MDOS). However, known MDOS on the basis of jalousie heliostats (JH) do not provide complete transmission of solar flux because of optical losses due to shading and obstruction of rays. Losses caused by shading and obstruction can reach 30% of incident solar radiation. In contrast to solar modules with mirror concentrator systems MDOS uses parallel ray bundle at the receiver and provides 100% for passing rays, as well as zero optical losses characteristic of known mirror deflecting optical systems. The mirror deflecting optical system can be used for trans mission of parallel solar flux to radiation receivers in hot houses, buildings and underground facilities.

Keywords: *electric power supply, photovoltaic plant, solar module, mirror deflecting optical system, jalousie heliostat.*

One of the trends of reduction of photovoltaic equipment cost is the use of photovoltaic modules with solar concentrators.

The purpose of this work is the development of the concentrator solar module with low optical losses and high uniformity of illumination at a radiation receiver, as well as with high specific capacity of the radiation receiver.

One of simple and effective methods of providing uniform distribution of concentrated solar radiation at the receiver surface is the use of mirror deflecting optical systems (MDOS). However, known MDOS on the basis of jalousie heliostats (JH) do not provide complete transmission of solar flux because of optical losses due to shading and obstruction of rays. Losses caused by shading and obstruction can reach 30% of incident solar radiation [1, 2, 3].

At Fig. 1 the diagram of the deflecting optical system of jalousie heliostats (JH) and ray path there in is shown.

A solar concentrate or has working surface 1 where on radiation 2 is falling, deflecting the optical system 3 with the surface of ray entry 4 and exit 5, height, width *l* and length L, comprising basic mirror reflectors 6 placed at the angle φ to the vertical of the working surface 1, and 1 and additional mirror reflectors 7 placed at the surface of exit 5 of the deflecting optical system 3 at the angle β_1. Basic mirror reflectors 6 are installed at the distance *a*, from each other.

The number of basic 6 and additional 7 mirror reflectors in the deflecting optical system 3 is 3 $n = \frac{L}{a}$. Let us denote the ray entry and exit angles to the basic mirror reflectors 6 in the deflecting optical system 3 by β_0 and β_1. The angles β_0 and β_1 are measured from the vertical to the working surface. The angle β_1 is chosen up on the condition of maximal deflection of reflected ray at the exit of the system at the distance $OE = 2a - \delta$ from the of ray entry line *AB* where δ is an infinitely all value providing total optical transparency of the deflecting optical system 3.

At Fig. 2 ray transmittance of rays β_0 from the basic mirror reflectors 6 from triangles BDN and DNK is as follows:

$$\Delta = d \cdot \mathrm{tg}\beta_0 \cdot \cos\varphi;$$

$$\Delta = d_1 \cdot \mathrm{tg}\beta_1, \qquad (1)$$

where *d*; d_1 – dimensions of the basic 6 and additional 7 mirror reflectors.

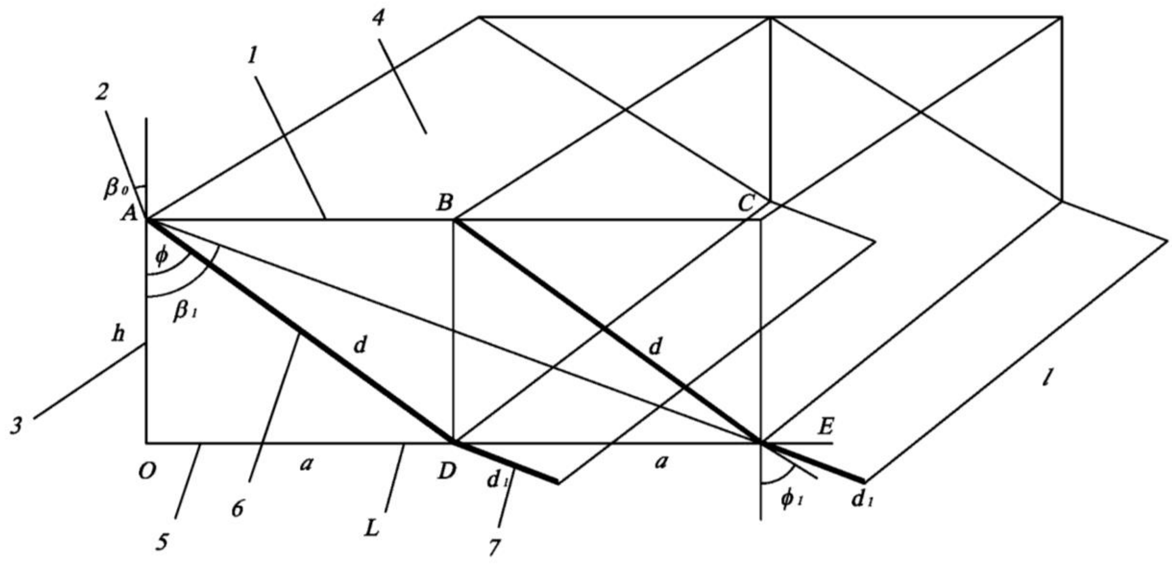

Fig. 1. The diagram of the deflecting optical system of the concentrator solar module and ray path therein (two-dimensional image)

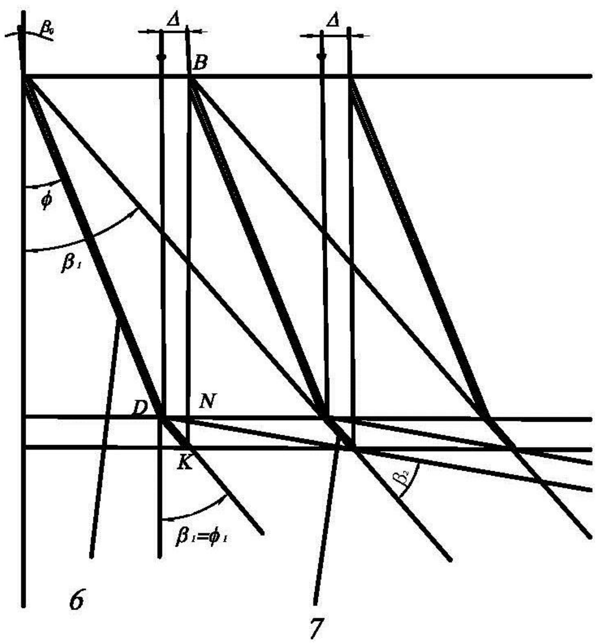

Fig. 2. Ray transmission in the in the deflecting optical system of the concentrator solar module

From (1) we obtain the relation for the width of additional mirror reflectors d_1:

$$d_1 = \frac{d \cdot \cos\varphi \cdot \sin\beta_0}{\sin(\beta_1 - \beta_0)}. \qquad (2)$$

The angle of ray exit β_2 for the β_0 in light rays is equal to:

$$\beta_2 = 2\varphi_1 - \beta_0 = 2\beta_1 - \beta_0. \qquad (3)$$

The installation of additional mirror reflectors 7 makes it possible to deflect by an angle β_2 the rays β_0, for which the deflecting optical system 3 comprising mirror reflectors 6 was transparent and could provide 100% reflection of all rays β_0 falling on the working surface 1 of the concentrator solar module.

In the concentrator solar module 3 the deflecting optical system 3 with width $B = QO$ creates flux of parallel rays with angles $\beta_1$and β_2, on the surface of exit 5, which come to the receiver 8 with width $A = OO_1$, installed along the path of β_1 and β_2 at the plane OO_1, perpendicular to the working surface 1 and passing through the lateral side 9 of the deflecting optical system 3.

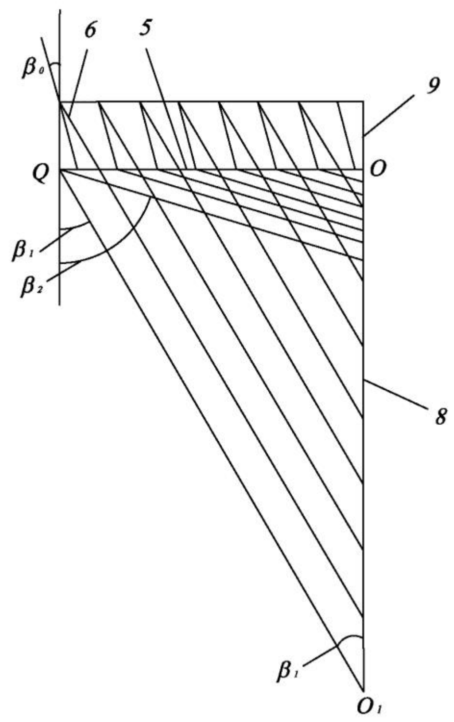

Fig. 3. Concentrator solar module and a single-ended receiver

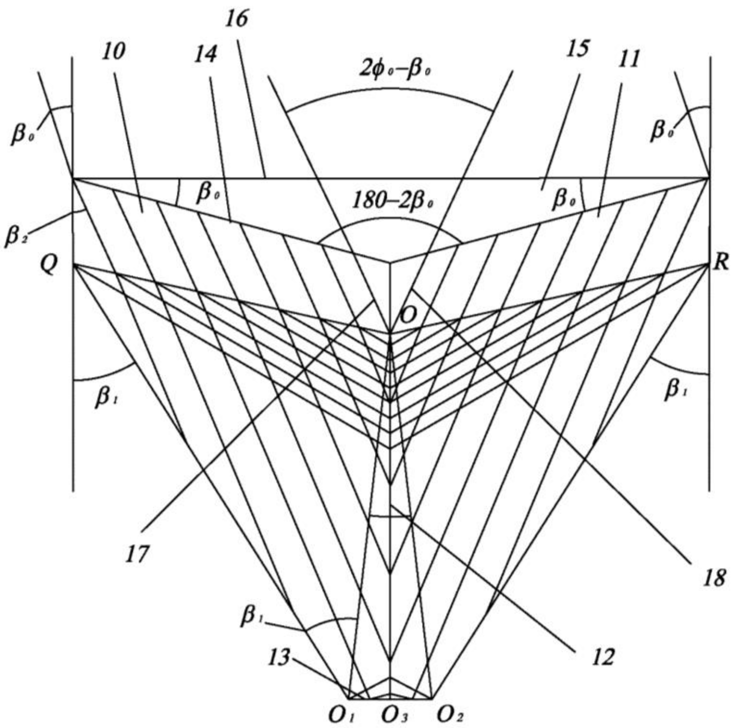

Fig. 4. The concentrator solar module comprising two deflecting optical systems with a common receiver

From the triangle QOO_1:

$$A = OO_1 = QO \cdot \mathrm{ctg}\beta_1 = B \cdot \mathrm{ctg}\beta_1. \qquad (4)$$

Concentration factor for deflecting optical system 3 QOO_1 with consideration to cosine losses at $\beta_0 \neq 0$ will amount to:

$$k_1 = \frac{QO \cdot \cos\beta_0}{OO_1} = \frac{B \cdot \cos\beta_0}{A} = \mathrm{tg}\beta_1 \cdot \cos\beta_0 . \qquad (5)$$

At Fig. 4 two refracting optical systems 10 QOO_1 and 11 ROO_2 have common two side receiver with size of $A \cos \beta_0$, installed in the symmetry plane 12 OO_3 of the solar module. The angle O_1OO_3 is equal to the angle O_2OO_1 and is equal to β_0.

The section $O_1O_3O_2O$ is made in the form of a mirror at reflector 13 with dimensions $2A \cdot \sin \beta_0$.

The planes of ray entry 14 and 15 are angled to the midsection plane 16 at the angles β_0. The angle between the planes of ray entry 14 and 15 is 180^o-$2\beta_0$, and the angle between the planes of basic mirror reflectors 17 and 18 in the two deflecting optical systems 10 and 11 is $2\varphi - 2\beta_0$.

The concentration factor of the solar module at Fig. 4 considering cosine losses is:

$$k_2 = \frac{QR \cdot \cos\beta_0}{OO_3} = 2\mathrm{tg}\beta_1 = 2k_1 . \qquad (6)$$

At Fig. 5 flat mirror reflector 19 is installed at the medial line I_1I_2 ΔQO_1O in parallel to the plane of ray exit 5 QO in such a way that ray QI_1 reflected from the mirror reflector 19 gets to the point O. The one-sided receiver 20 with dimensions $A = OI_2$ is installed in the plane OI_2. Concentration factor of the solar module at Fig. 5 considering cosine losses is:

$$k_3 = \frac{QO \cdot \cos\beta_0}{OI_1} ; \qquad (7)$$

$$A = OI_1 = IG = QG \cdot \mathrm{ctg}\beta_1 ; \qquad (8)$$

$$QO = 2QG ; \qquad (9)$$

$$k_3 = \frac{2QG \cdot \cos\beta_0}{QG \cdot \mathrm{ctg}\beta_1} = 2\mathrm{tg}\beta_1 \cdot \cos\beta_0 . \qquad (10)$$

At Fig. 6 the concentrate or solar module has two symmetrically installed reflecting optical systems 10 and 11 and two mirror reflectors I_1I_2 19 and I_3I_4 21 installed at the medial line ΔQO_1O and ΔOQO_2. The mirror reflectors 19 and 21 are connected by the flat mirror reflector 22 with dimensions of $2A \cdot \sin\beta_0$, and the receiver 23 with dimensions of $A \cdot \cos\beta_0$ is installed in the symmetry plane OO_4.

The concentration factor of the concentrator solar module considering cosine losses is:

$$k_4 = \frac{QR \cdot \cos\beta_0}{OO_4} = 4\mathrm{tg}\beta_1. \quad (11)$$

The concentrator solar module operates in the following way:

Solar radiation (Fig. 3) comes to the mirror reflector 6 at the entry angle β_0 and is reflected at the angle β_1. The mirror reflector 13 at the Fig. 4, 19 the Fig. 5, mirror reflectors 19, 21, 22 at Fig. 6 reflect parallel rays with angles β_1 and β_2, coming from the deflecting optical system 3 to the radiation receiver. The mirror reflector 13 at Fig. 4, 19 at Fig. 5, mirror reflectors 19, 21, 22 at Fig. 6 reflect parallel rays with angles β_1 and β_2, coming from deflecting optical system 3 to the radiation receiver.

We shall give several examples of concentrate or solar module configurations:

The deflecting optical system at Fig. 3 comprises basic mirror reflectors 6 with dimensions of d = 50 mm, l = 1000 mm and additional mirror reflectors 7 with dimensions of d_1 = 6.86 mm, l = 1000 mm. The tilt angle of the basic mirror reflectors 6 to the vertical plane is $\varphi = 22.5^o$, the angle of ray entry is $\beta_0 = 5.4^o$, the angle of ray exit is $\beta_1 = 39.6^o$, the tilt angle of additional mirror reflectors 7 is $\varphi_1 = \beta_1 = 39^o$, the angle of ray exit is $\beta_2 = 73.8^o$. The distance between the basic mirror reflectors 6 is 6 a = 19.13 mm, transmission Δ = 4.37 mm.

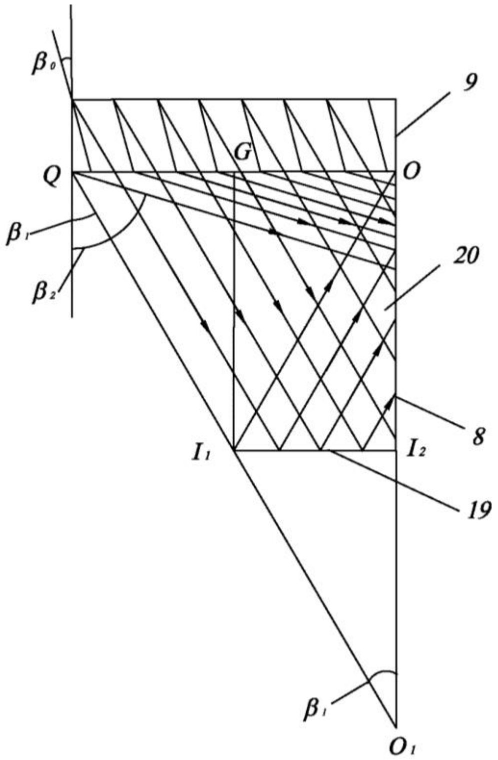

Fig. 5. Concentrator solar module comprising a flat mirror reflector parallel to the module working module

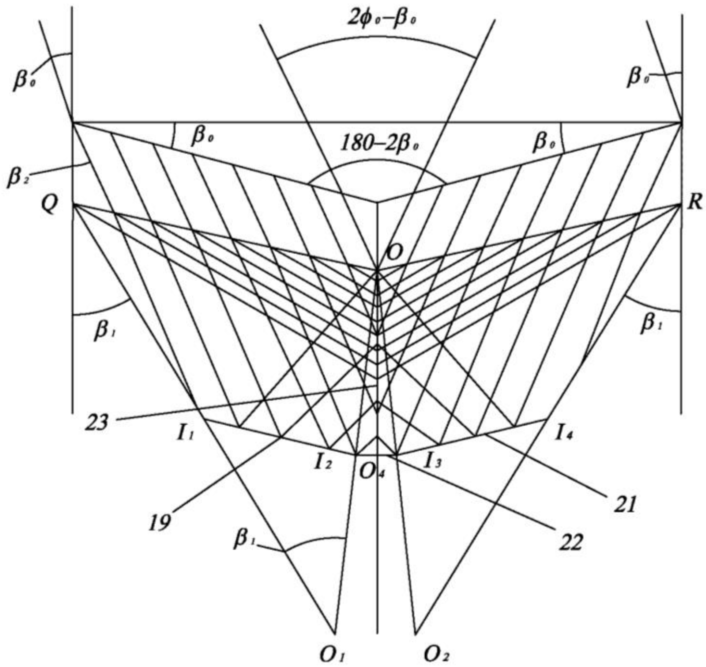

Fig. 6. Concentrator solar module comprising two deflecting optical systems with a common receiver and two flat mirror reflectors parallel to the surface of rays exit

Table 1

Total area of mirror reflectors per 1m² of the module area, m²/m²

φ	10°	20°	22.5°	25°	30°	35°	40°
S_{30}	5.76	2.92	2.61	2.37	2	1.74	1.56

Concentration factor for the receiver at Fig. 6 will amount to k_4 = 3.31.

Let us determine the area of the basic mirror reflectors 6 S_{30} on the surface of entry 4 of the deflecting optical system 3 with dimensions for a concentrate or solar module 1×1 m. The area of one mirror reflector 6 with 1 m length:

$$S_1 = 1 \cdot d\text{, m}^2, \quad (12)$$

where d - width of the reflector, m.

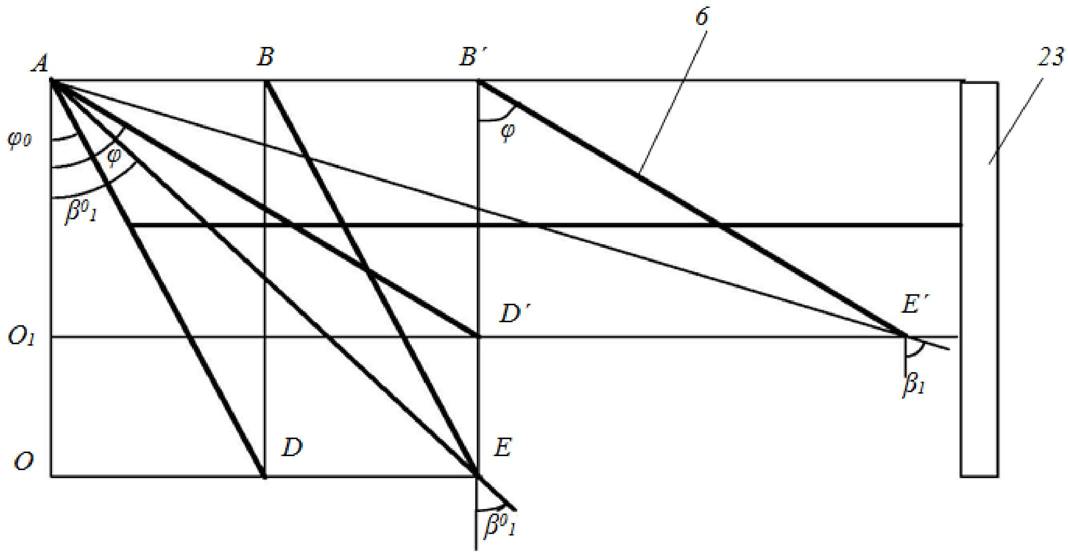

Fig. 7. Optical system and ray path in the concentrator solar module

The distance between mirror faceted reflector $a = d \cdot \sin\varphi$ (Fig. 1). The number of mirror reflectors on the surface of a module with 1×1 m:

$$n = \frac{1}{d \sin\varphi}, \qquad (13)$$

$$S_{30} = n \cdot S_1 = \frac{1}{\sin\varphi} = \mathrm{cosec}\varphi \qquad (14)$$

The total area of mirror reflectors does not depend on the area of individual mirror reflectors and their number but is determined by the angle of the reflector φ to the vertical plane:

$$\lim_{\varphi_0 \to 0} S_{30} = \infty;\ At\varphi = 90^{\circ} S_{30} = 1\ \mathrm{m}^2.$$

In Table 1 the total area of mirror reflectors per 1 m^2 of the module area is given, m^2/m^2.

For practical applications it is important to use jalousies where in the mirror length remains permanent where in the mirror tilt angle φ is changed, and the distance *a* between the mirrors is changed. Let us fix the value *AD* = *d* - the width of the mirror reflector and φ_0 – the initial tilt angle of the mirror reflect or at which points Band Dare in the same vertical plane (Fig. 7).

Figure 7 shows that at $\varphi > \varphi_0$, *d* = const, *B*′ is in the same plane with the point *D*′. At Fig. 7 there are shown the positions of the second mirror *B*′*E* and the path of reflected ray *AE*′ at *d* = const, at which *B*′ is in the same plane with *D*′.

At $\varphi > \varphi_0$, to fit the reflected ray β_1 yinto the size $D'E' = a$, β_0 and *a* must be increased.

The angle of reflected ray exit at Fig. 7:

$$\beta_1 = \mathrm{arctg}(2\mathrm{tg}\varphi) = 2\varphi - \beta_0\,. \qquad (15)$$

The distance between the miniature reflectors at the initial angle φ_0 of mirror reflectors:

$$a_0 = d \cdot \sin\varphi_0\,. \qquad (16)$$

At arbitrary tilt angle φ of mirror reflectors

$$a = d \cdot \sin\varphi. \qquad (17)$$

From (16) and (17) follows:

$$a = a_0 \frac{\sin\varphi}{\sin\varphi_0}\,. \qquad (18)$$

At Fig. 7 the deflecting optical system has the device 23 for changing distance between mirror reflectors and the tilt angle of the mirror reflectors.

In contrast to solar modules with mirror concentrator systems MDOS uses parallel ray bundle at the receiver and provides 100% for passing rays, as well as zero optical losses characteristic of known mirror deflecting optical systems.

The mirror deflecting optical system 3 can be used for trans mission of parallel solar flux to radiation receivers in hot houses, buildings and underground facilities.

Conclusions

1. The deflecting optical system (DOS) providing 100% transmission of reflected rays at the exit surface has been developed.

2. Optical calculation of ray path is given and the concentration factor of solar modules with a mirror deflecting optical system and various types of concentrators is considered.

References

1. *Teplyakov D.I., Tver'yanovich E.V.* Linear jalousie heliostats solar powers: cosine effects and between jalousie heliostats effects // Solar technology, 1993, №4, p. 54-58.
2. *Teplyakov D.I., Tver'yanovich E.V.* Linear jalousie heliostats solar powers: loss of radiation // Solar technology, 1993, №5, p. 54-57.
3. *Teplyakov D.I., Tver'yanovich E.V.* Linear jalousie heliostats solar powers: loss of energy production // Solar technology, 1993, №6, p. 63-70.

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ИССЛЕДОВАНИЕ ЗЕРКАЛЬНЫХ ОТКЛОНЯЮЩИХ ОПТИЧЕСКИХ СИСТЕМ ДЛЯ СОЛНЕЧНЫХ МОДУЛЕЙ С КОНЦЕНТРАТОРАМИ

Д.С. Стребков, А.Е. Иродионов, Н.С. Филиппченкова
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Разработана отклоняющая оптическая система, обеспечивающая 100%-ное пропускание отраженных лучей на поверхности выхода. Проведен оптический расчет хода лучей и определен коэффициент концентрации солнечных модулей с зеркальной отклоняющей оптической системой и различными типами концентраторов. Одним из направлений снижения стоимости изделий фотоэлектрической энергетики является использование фотоэлектрических модулей с концентраторами солнечного излучения. Целью работы является разработка солнечного модуля с концентратором, имеющего низкие оптические потери и высокую равномерность освещения на приемнике излучения, а также повышенную удельную мощность приемника. Одним из простых и эффективных методов создания равномерного распределения концентрированного солнечного излучения на поверхности приемника является использование зеркальных отклоняющих оптических систем (ЗООС). Однако известные ЗООС на основе жалюзийных гелиостатов (ЖГ) не обеспечивают полного пропускания солнечного потока из-за оптических потерь на затенение и блокировку лучей. Потери на затенение и блокировку могут достигать 30 % поступающего солнечного излучения. В отличие от солнечных модулей с зеркальными концентрирующими системами ЗООС использует параллельный пучок лучей на приемнике и обеспечивает 100%-ную прозрачность для проходящих лучей и отсутствие оптических потерь, свойственных известным зеркальным отклоняющим оптическим системам. Зеркальная отклоняющая оптическая система может быть использована для передачи параллельного потока солнечной энергии на приемники излучения в теплицах, зданиях и в подземных сооружениях.

Ключевые слова: электроснабжение, фотоэнергетическая станция, солнечный модуль, зеркальная отклоняющая оптическая система, жалюзийный гелиостат.

Литература

1. *Тепляков Д.И., Тверьянович Э.В.* Линейные жалюзийные гелиостаты СЭС: косинусные и межжалюзийные эффекты // Гелиотехника. 1993. №4. С. 54-58.
2. *Тепляков Д.И., Тверьянович Э.В.* Линейные жалюзийные гелиостаты СЭС: потери радиации // Гелиотехника. 1993. №5. С. 54-57.
3. *Тепляков Д.И., Тверьянович Э.В.* Линейные жалюзийные гелиостаты СЭС: потери выработки энергии // Гелиотехника. 1993. №6. С. 63-70.

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THE DEVELOPMENT AND TESTING OF FOLDING, SECTIONAL AND FLEXIBLE SOLAR MODULES

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Compact design solar photovoltaic modules solve a problem of charging of portable electronic devices in lack of network power supply. The optimized designs both folding two-fold, and sectional compact figurative solar modules are considered. They were tested under natural conditions for work in conjunction with modern gadgets. Also flexible, thin and lightweight solar modules with standard layout and with design execution are provided. The modules are suitable for charging lead-acid gel batteries with a voltage of 12 V, and modern lithium battery with a voltage of 3,7 V. Along with universality modules possess the small sizes, compactness, a sectional design with possibility of scaling, and also have the small weight and possibility of an arrangement them on a curvilinear surface.

Keywords: *electric power supply, sectional and folding solar modules, flexible solar module, gadgets.*

The problem of charging of portable electronic devices when there is no possibility of connection to the power grid is rather urgent for tourists, cottagers and travelers. Solar modules of compact design are used for the solution of this problem. The article presents new design of such modules with optimized technical, design and electrical parameters.

Solar modules of compact design 505 and 505C (Fig. 1 and 2) are meant for power supply to compact mobile electric devices with USB standard (5 V, 0.5 A), as well as for assembling sections into batteries. As an output connector it is suggested to apply a standard USB-socket because of a wide use of this standard for charging mobile electronic devices. For connection to the USB-bus interface a four-wire cable is used – two wires (twisted pair) in differential connection are used for receiving and transmitting data, and two wires – for power supply to a peripheral device. Due to integrated power supply lines, USB allows to connect peripheral devices without own power source (maximal intensity of current consumed by the device through power supply lines of the USB bus must not exceed 500 mA, of the USB 3.0 – 900 mA).

Electric power is supplied to small-size electric equipment in autonomous mode via direct connection to a solar module without adaptors and

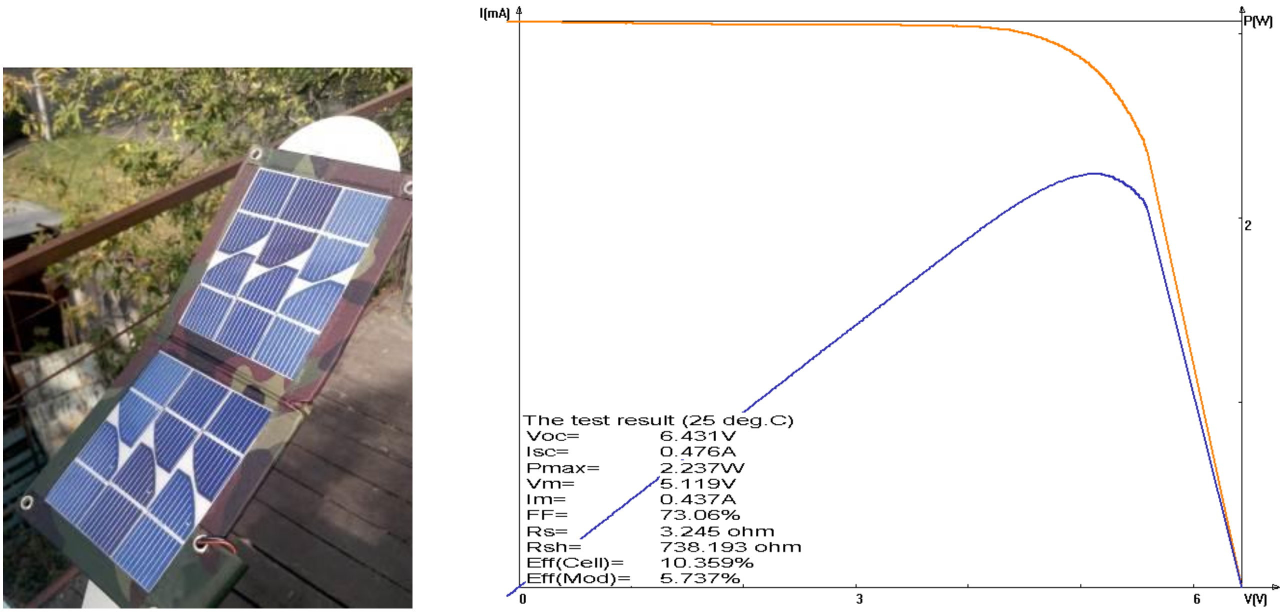

Fig. 1. Module 505 with 12 cells and its volt-ampere characteristic

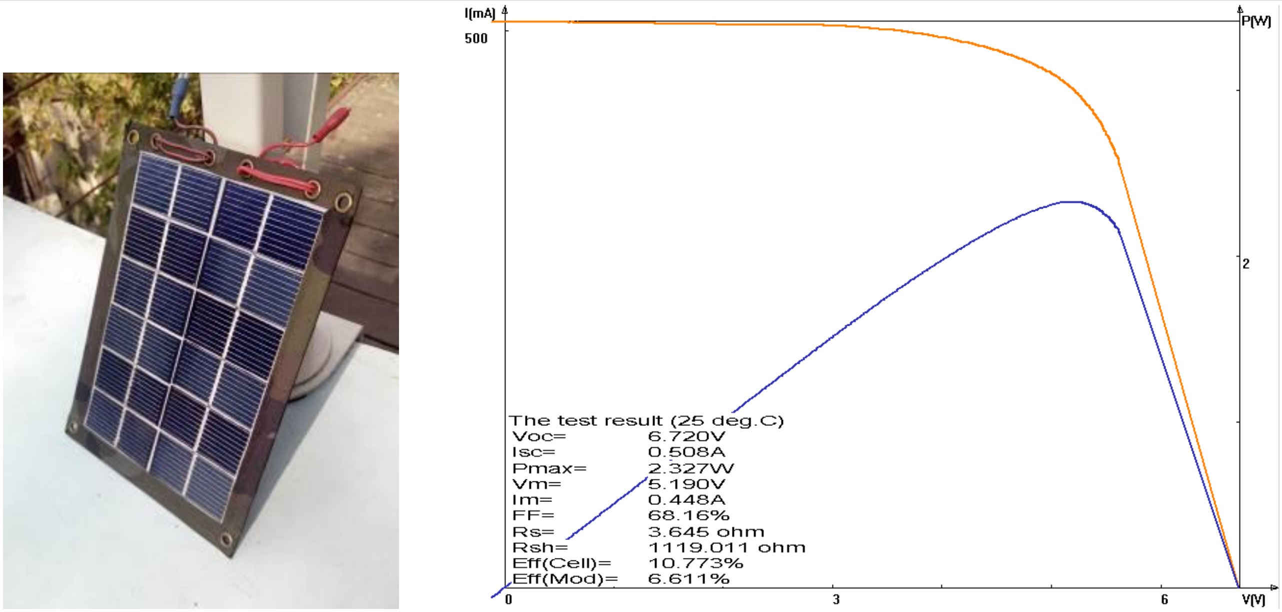

Fig 2. Module 505C with 12 cells and its volt-ampere characteristic

stabilizers. The design of a sectional module is unified (Fig. 2), thus expanding the range of potential customers when section commutation is serial-parallel. Folding and sectional solar modules are made with varying exterior design, standard USB parameters and the possibility of sections commutation.

In their electric characteristics, the size and commutation of module solar cells are adapted to the widespread USB standard of electric power supply without any additional stabilizers and converters (when stabilizers and converters are used, the range of powered devices is expanded). There is the possibility of direct power supply to various electric devices via USB meeting the most demanding electric requirements. In series-parallel connection, capacity is enlarged, taking into account required output electric parameters. Easily detached connectors, eyelets and magnets make operation of the modules much easier. Varying external design and better esthetic characteristics will satisfy the most demanding consumers.

The modules are composed of commutated laminated solar cells with size adapted to USB power supply standard, and hard substrate in order to exclude solar cells bending and breaking. The sectional modules completing parts include a USB plug, a socket with wires and clams for 5 V 0.5 – 1 A and 5 V 3 A, a 12 V jumper for power supply and a set of alligator clips.

The parameters of volt-ampere characteristic of the modules (ВАХ) ($P_{макс}$, U_{xx} , $I_{кз}$, W, m, η) were measured with the use of a PICOSOLAR solar radiation simulator with a single long-pulse flash. In testing the indicators U_{xx} , $I_{кз}$, U_p, $U_з$ in natural conditions, the level of the device charge is investigated.

The following measuring devices and equipment were used during research:

- the solar radiation simulator with a single long-pulse flash (1000 W/м^2, 1.5 AM) PICOSOLAR;
- the electronic multimeter of the MY-64 type (a voltmeter, a thermometer);
- the magnetoelectric amperometer of the M-1104 type;
- a metal rule for measuring linear dimensions of photovoltaic cells groups and the solar module dimensions.

After measuring the modules volt-ampere parameters with the use of the solar radiation simulator and research of basic volt-ampere parameters, the parameters of the solar modules were measured under natural solar radiation (the solar module is oriented perpendicularly to solar rays).

The models researched

1. *The model 505* (Fig. 1) (*prototype – 12 cells by 1/8 part of the 125 x 125 mm cell*)**,** USB standard (5 V, 0.5 A, 2.5 V)**,** USB –plug at the outlet, the module dimensions in a folded state - 130 mm x 110 mm, weight – 120 g.

The following electric devices have been tested for operational performance and the speed of charging by the 505 module:

1) the ventilator with a drive from the USB port works (rotates) from the solar panel in nominal mode;

Table 1

Charging of assembly of accumulator batteries by the 505 C module

Time, hours:min	12:20	13:30	14:10	15:30	16:30	17:30	18:00
Uxx, V (no-load voltage)	5.90	6.20	5.97	5.45	5.97	6.15	5.33
Iкз, A (short circuit current)	0.37	0.38	0.39	0.05	0.38	0.36	0.05
Up, V (operating voltage)	5.60	5.80	5.66	5.35	5.74	5.81	5.35
Uз, V (battery charge)	4.90	5.16	5.22	5.32	5.35	5.37	5.38

Table 2

Charging of the boost Li-ion accumulator by the 505 C solar module

Time, hours:min	11:20	12:00	13:30	15:00	16:00	17:00	11:20	12:45
Uxx, V(no-load voltage)	5.70	5.90	5.85	5.87	5.82	6.04	5.85	5.90
Iкз, A (short circuit current)	0.30	0.40	0.41	0.38	0.39	0.35	0.37	0.39
Up, V(operating voltage)	3.80	3.90	4.03	4.08	4.12	4.12	4.07	4.17
battery charge, points 1-4	1	1	1	1	2	2	2	2

2) the lamp with one LED and power supply from the USB-port works (shines) from the solar panel in nominal mode;

3) the lamp with 8 LED and power supply from the USB-port works (all 8 LED shine) from a solar panel in nominal mode;

4) the compact speaker with power supply from USB is steadily charging from the solar panel;

5) the mobile telephone Nokia 2700 (accumulator 3.7 V, 1020 mAh) – was completely charged in 2 hours 50 min;

6) the mobile telephone Megaphone MINIFON (accumulator 3.7 V, 450 mAh) was completely charged in 1 hours 20 min;

7) the tablet computer Explay Informer 701 (Li-Pol accumulator 3.7 V, 3000 mAh) (the tablet computer is switched on) – 80 % charge in 6 hours 40 min;

8) the mobile telephone Motorola Droid Razr M (Li-Ion accumulator 3.7 V, 2000 mAh) (the telephone is switched on) – 21 % charge in 2 hours 30 min.

2. *The model 505 C* (Fig. 2) (*prototype – 12 cells by 1/8 part of the 125 x 125 mm cell*)**,** USB standard (5 B, 0.5 A, 2.5 W). A USB connector at the outlet, section dimensions: 210 mm x 143 mm, possible number of section in the panel – 1 – 12 and more (2.5 – 30 V and more), weight – 140 g.

The following electric devices have been tested for operational performance and the speed of charging by the 505C module:

Points 1) – 4) are identical with those in testing the 505 module;

5) the assembly of boost battery comprising 4 accumulator batteries of the AA type (1.2 V 2 300 mA each). The device charge and electrical indicators are given in Table 1;

6) the boost Li-ion accumulator QUMO PowerAid 6600 (5 B 6600 mA). The device charge and electric indicators are given in Table 2.

All the above-mentioned electric devices were steadily charged from the developed modules, however, for more energy consuming devices (Iphone 4, etc,) the solar modules should be improved via optimization of output voltage and the schemes of the output USB connector. Of all developed schemes of output USB connector we have chosen the most flexible and efficient with optimal additional resistance, providing stable charging all the earlier researched devices, as well as more energy-consuming equipment. The same scheme is used in all output USB-connectors of the compact solar modules.

3. *The model 505/505C* (Fig. 3 and 4) (*prototype - 10 cells by 1/6 part of the 125 x 125 mm cell*)**,** USB standard (5 V, 0.5 A, 2.5 V), a USB connector at the outlet, The module dimensions in a folded state: 140 mm x 130 mm, weight – 140 and 180 g.

The following electric devices have been tested for operational performance and charging rate by the 505C module with 10 elements by 1/6 part of the 125 x 125 mm cell:

Points 1) – 4) are the same as in testing the 505 module with 12cells;

5) the mobile telephone Nokia 2700 (accumulator 3.7 V, 1020 mAh) is charged steadily;

6) the mobile telephone Megaphone MINIFON (accumulator 3.7 B, 450 mAh) is charged steadily;

7) the tablet computer Explay Informer 701 (accumulator Li-Pol 3.7 V, 3000 mAh (the tablet computer is switched on) is charged steadily;

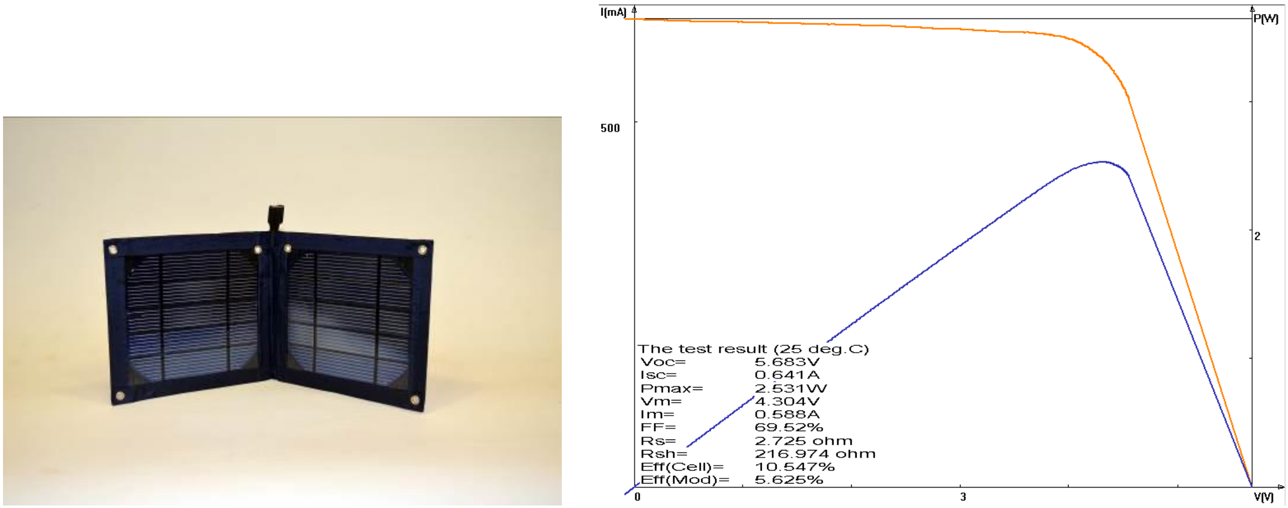

Fig 3. Module 505 with 10 cells and its volt-ampere characteristic

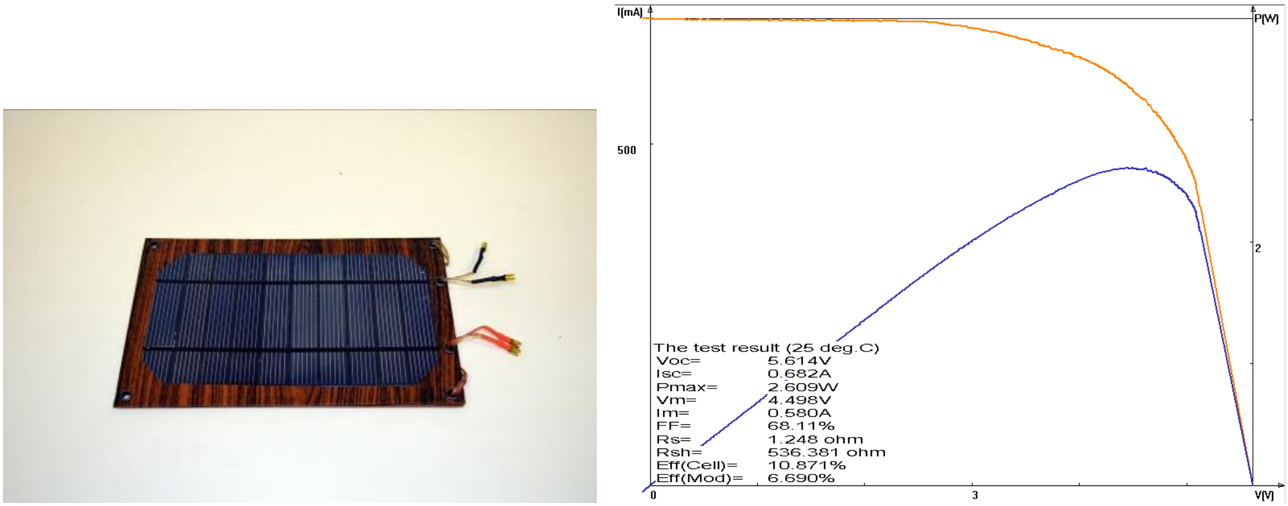

Fig. 4. Module 505C with 10 cells and its volt-ampere characteristic

8) the assembly of boost accumulator of 3 accumulator batteries of the AA type (1,2 V 2300 mA each)) is charged steadily;

9) the boost Li-ion accumulator QUMO PowerAid 6600 (5 B, 6600 mA) is charged steadily;

10) the mobile telephone Iphone 4 (accumulator Li-Ion 3.7 V, 1420 mAh) (the telephone is switched on). The device charge and electric indicators are given in Table 3;

11) the mobile telephone Motorola Droid Razr M (Li-Ion accumulator 3.7 V, 2000 mAh) (the telephone is switched on). The device charge and electric indicators are given in Table 4.

4. *The model 505* (Fig. 5) (*prototype – 10 cells by 1/4 part of the 125 x 125 mm cell*), USB standard (5 V, 0.5 A, 2.5 W), USB-connector at the outlet, the module dimensions in folded state: 180 mm x 150 mm, weight – 230 g.

The following electric devices have been tested for operational performance and charging rate by the 505C module with 10 cells by 1/6 part of the 125 x 125 mm cell:

Points 1) – 9) are the same as in testing the 505 C module with 10 cells by 1/6 part of the 125 x 125 mm cell;

10) the mobile telephone Iphone 4 (Li-Ion accumulator 3.7 V, 1420 mAh) is charged steadily;

11) the mobile telephone Motorola Droid Razr M (Li-Ion accumulator 3.7 V, 2000 mAh) is charged steadily;

12) the boost Li-ion accumulator QUMO PowerAid 6600 (5 V, 6600 mA). The device charge and electric indicators are given in Table 5.

While using solar panel with expanded area of solar cells (1/4 part of the 125 x 125 mm cell), increase of speed of charging of all the electric

Table 3

Charging of Iphone 4 by the 505 C solar module with 10 cells by 1/6 part of the 125 x 125 mm cell

Time, hours:min	11:00	13:00	14:30	16:30	18:00	13:00	14:20	15:30	16:00
Uxx,V ((no-load voltage)	5.01	5.12	5.04	5.10	4.27	5.38	5.04	5.14	5.01
Iкз, A (short circuit current.)	0.39	0.2	,43	0.42	0.02	0.57	0.57	0.55	0.53
Up, V (operating voltage)	4.69	4.64	4.75	4.60	4.26	4.60	4.76	4.83	4.98
Charge state, relative units	0.09	0.35	0.51	0.54	0.74	0.52	0.82	0.94	1

Table 4

Charging of Motorola Droid Razr M 4 by the 505 C solar module with 10 cells by 1/6 part of the 125 x 125 mm cell

Time, hours:min	11:30	12:30	13:30	15:30	17:00	18:00	18:30
Uxx, V(no-load voltage)	5.40	4.99	4.90	4.87	4.96	4.99	4.95
Iкз, A (short circuit current)	0.45	0.50	0.50	0.49	0.45	0.35	0.24
Up, V(operating voltage)	4.51	4.48	4.47	4.47	4.46	4.43	4.39
Charge state, relative units	0	0.20	0.30	0.52	0.70	0.78	0.81

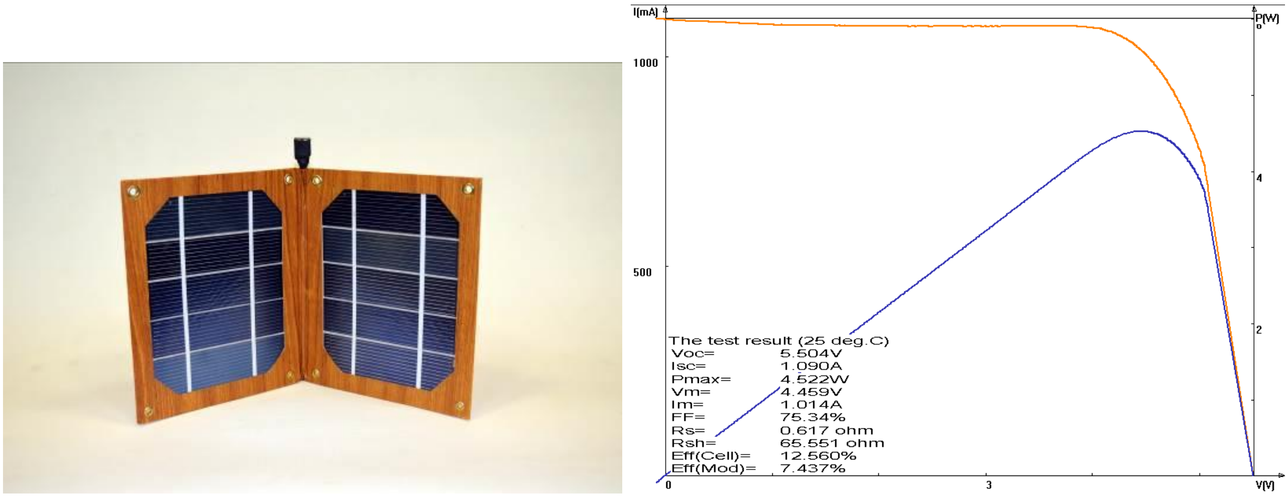

Fig. 5. Module 505 with 10 cells and its volt-ampere characteristic

Table 5

Charging of the Li-ion accumulator QUMO PowerAid 6600 by the solar module 505 with 10 cells by 1/4 part of the 125 x 125 mm cell

Time, hours: min	11:30	12:30	13:30	15:30	17:00
Uxx, V (no-load voltage)	5.23	5.08	5.00	4.97	4.99
Iкз, A (short circuit current)	0.25	0.75	0.74	0.72	0.67
Up, V (operating voltage)	3.95	4.48	4.45	4.45	4.48
Charge state, points 1-4	1	2	2	2	2

devices is observed, as the module current is increased in proportion to the solar cells area.

In the course of compact modules development, solar modules of two types with various external design have been made – folding clamshell modules and single sectional modules. In order to reduce charging time and increase output current, a module with 10 solar cells by ¼ of the 125 x 125 mm cell with short circuit current about 1 A was suggested. For further increase of current or voltage the sectional modules are connected in series, in parallel or in series/parallel.

Alongside with folding and sectional solar modules, flexible transparent solar modules with

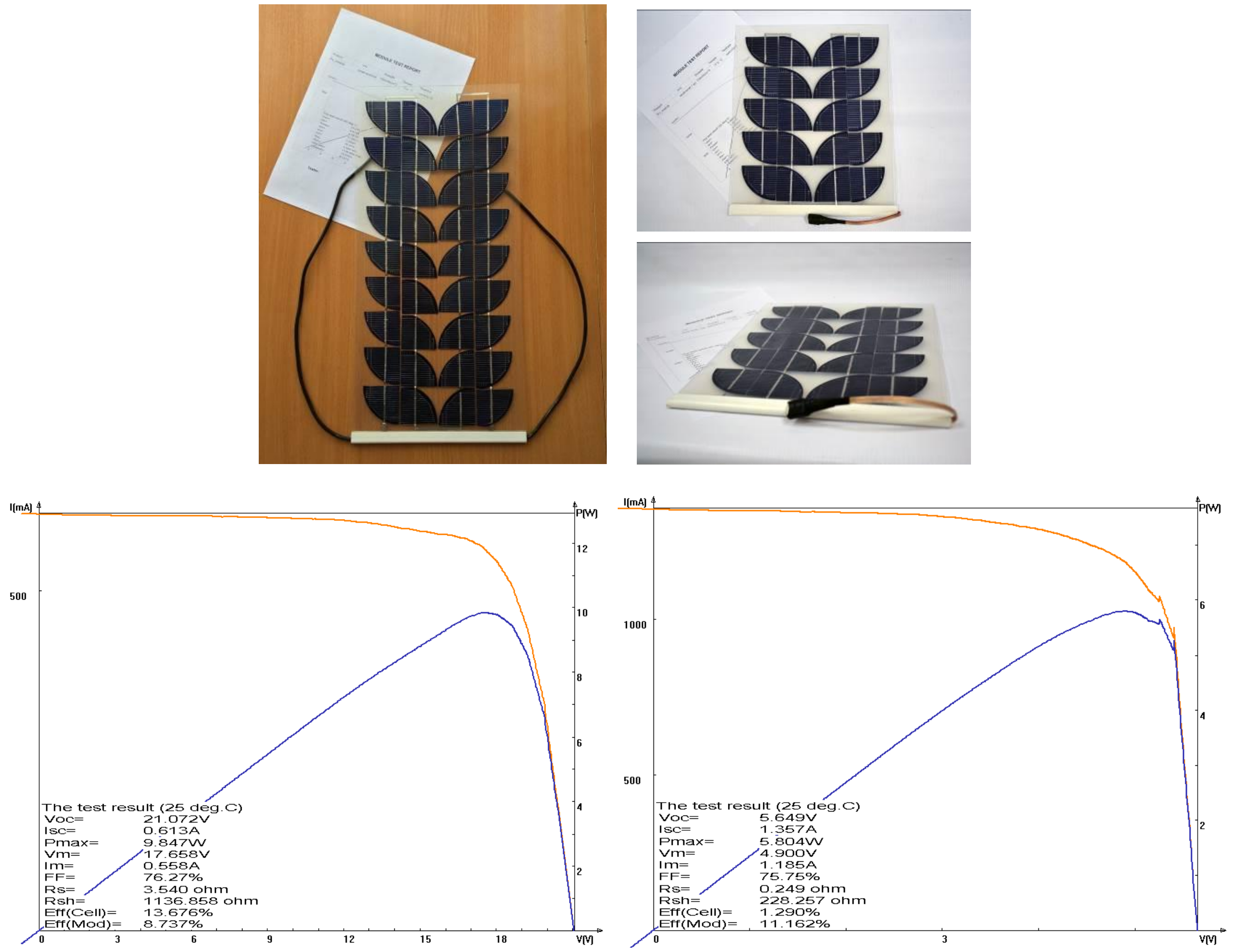

Fig. 6. Flexible transparent solar modules and their volt-ampere characteristics, with 17 V operating voltage (to the left) and 5 V operating voltage (to the right) designed for AGM (12 V) and lithium (3.7 V) accumulators

various parameters and cells configuration have been developed and fabricated. For research of experimental models and the use inside the premises, solar modules with unusual design and 12 V and 6 V voltage have been developed (Fig. 6 to the left and to the right). Two various indicators of voltages are related to the use of 12 V AGM accumulators and 3.7 V boost lithium accumulators of mobile devices for which operating voltage about 5 V are required, as in connection to the USB port.

The line of flexible solar modules also includes three types of standard 12 V modules – with operating current about 4 A, 2 A and 1 A depending on the area of the cell (Fig. 7). Optimal number and thickness of adhesive and laminating films allows to reliably protect solar cells both from external actions and makes it possible to impart bending form to modules, which is widely used in architecture and transport. All flexible modules offer the original solution of outlet of the MC 4 plugs, which allowed to exclude the use of a matchbox without sacrificing impermeability, and impart flat, even and thin form to the modules.

Therefore, considering the developed solar modules, it can be concluded that they are suited for charging of a wide range of energy-consuming devices. The possibilities of recharging the developed solar modules are not limited by AGM 12 V accumulators – with their use it is possible to successfully charge both portable handheld devices with lithium accumulators and boost lithium accumulators with 3.7 voltage. Alongside with flexibility, the modules are characterized by small size, co,mpactness, sectionability with the possibility of scaling, as well as by low weight and the possibility to place them at curved surface.

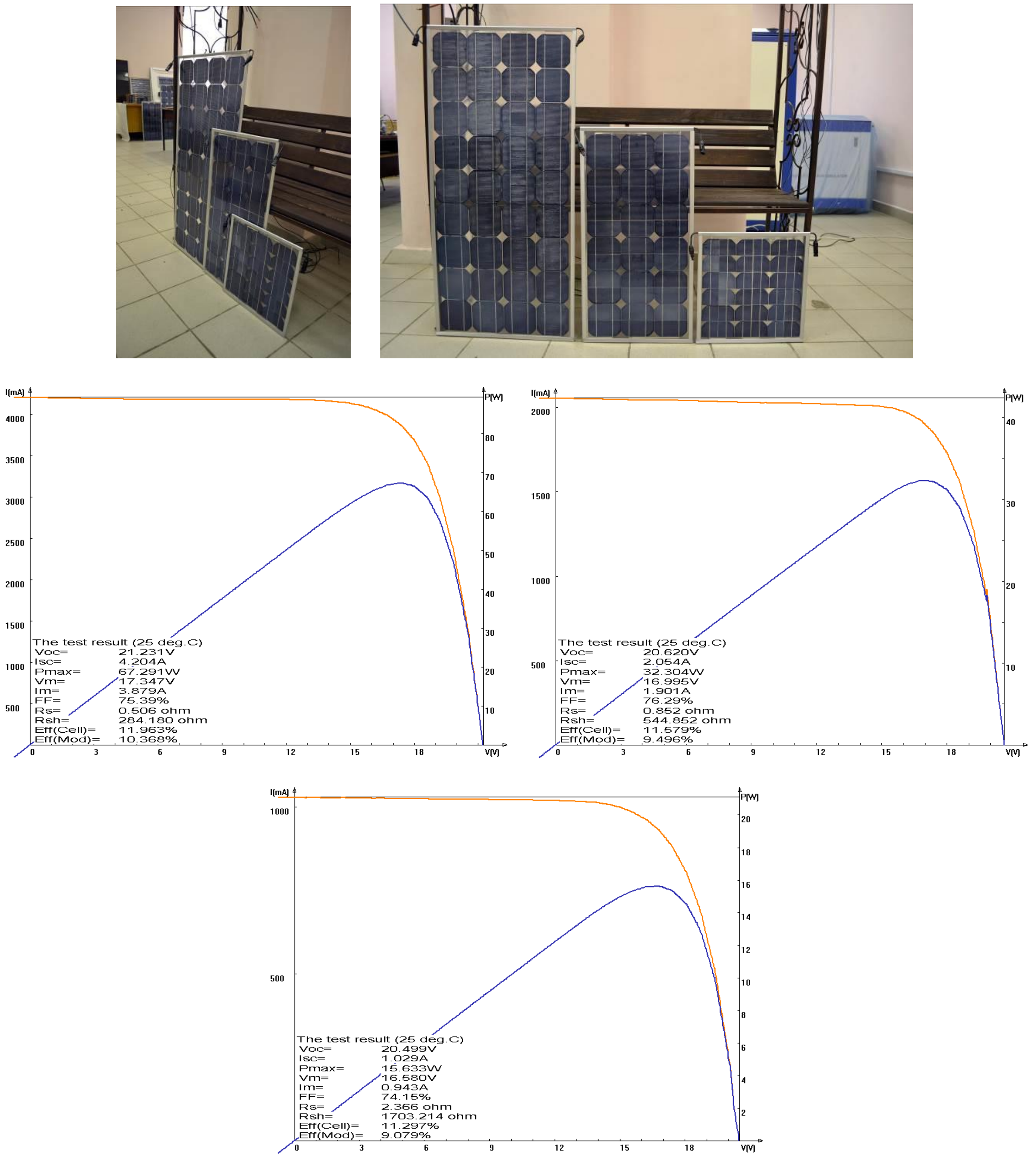

Fig. 7. Flexible transparent solar modules of three form factors with 17 V operating voltage, operating current 4 A, 2 A and 1 A and their volt-ampere characteristic

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РАЗРАБОТКА И ИСПЫТАНИЕ СКЛАДНЫХ, СЕКЦИОННЫХ И ГИБКИХ СОЛНЕЧНЫХ МОДУЛЕЙ

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Солнечные фотоэлектрические модули компактной конструкции решают проблему зарядки переносных электронных устройств в отсутствии сетевого электроснабжения. Рассмотрены оптимизированные конструкции как складных двустворчатых, так и секционных компактных переносных солнечных модулей. Проведены испытания их работы в натурных условиях совместно с современными гаджетами. Также представлены гибкие, тонкие и легкие солнечные модули, как стандартной компоновки, так и в дизайнерском исполнении. Модули пригодны как для зарядки свинцовых гелевых аккумуляторов с напряжением 12 В, так и литиевых современных аккумуляторов с напряжением 3,7 В. Наряду с универсальностью модули обладают малыми размерами, компактностью, секционностью с возможностью масштабирования, а также имеют малый вес и возможность расположения их на криволинейной поверхности.

Ключевые слова: электроснабжение, секционный и складной солнечные модули, гибкий солнечный модуль, гаджеты.

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THE REGENERATIVE WATER-AIR HEAT EXCANGER

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The article describes an energy-efficient scheme for using solar energy in district heat supply systems reducing capital and operation costs and to a great extent replacing conventional heat sources. The versatility of systems using this scheme provides their multi-zone applications: from cottages to heating industrial premises and greenhouse complexes. The principle of the suggested scheme is as follows: a solar collector is attached through heated medium is to an air duct and to antifreeze circulating in the contour tank – accumulator – solar collector. The method of intensification of heat exchange due to the use of intensifiers of hole type. Such intensification efficiency is determined by calculation of daily heat absorption of two-dimensional air collector with intensification by holes and without it, as well as economic effect of the implementation of the suggested method. In accordance with calculations, the suggested scheme of heating and intensification of heat exchange on the surface of integrated collector can provide significant economic effect if implemented.

Keywords: *solar collector, heat-carrying medium, design, simulation system, experiment, consumer, control, heat, energy saving, load optimization.*

The strategic aim of state energy policy in the field of development of rational fuel-power balance is the optimization of production structure, domestic consumption and export of fuel-power resources with due consideration of requirements of energy security, economical and energy efficiency and improvement of Russia foreign economical performance. It should be considered that the overriding priority of the «Energy Strategy of Russia for the Period until 2030» is the growth of importance of renewable energy sources in satisfying public energy demands. Without implementation of technologies that can drive out organic fuel from the country fuel balance, It is impossible to fulfill basic provisions of the Strategy envisaging maximum efficient use of natural energy resources and potential of power-generating sector for steady economic growth and the improvement of the quality of life of the RF population. For most RF regions the most promising of renewable energy sources is solar energy.

The Heat Supply Scheme

So far a wide range of various schemes of solar energy use in hot water supply and heating systems has been developed. Heating premises with hot air using heat sources of various types in many cases allows to considerably reduce capital and operational costs. Solar heating with the use of collectors of various types will make it possible to considerably increase such systems efficiency and to expand replacement of conventional heat sources. In such systems, depending on temperature mode, water or air are heated or combined heating is applied - water for hot water supply and air for heating. As our final objective is heating air in premises, it is exactly these complexes that allow to obtain maximum efficiency, excluding all intermediary processes and conversion. As energy source heat of combusted fuel, as well as heat generated by solar collectors, can be used [1].

The versatility of such systems determines their multi-zone application - from cottages to heating huge industrial premises and greenhouses complexes.

The advantages of air heating are as follows [2].

1. Heat economy - heat is generated directly in heated premises.

2. Improved living conditions in premises, as air heating by 40-70 ^0C is sufficient for positive pressure ventilation.

3. Fast response as the hot air heating system makes it possible to provide full heating of premises for 1.5-2 hours.

4. The absence of intermediate heat-carrying medium, that allows to exclude the construction of water heating systems. In winter season the risk of defreezing the system is excluded.

5. High degree of automation allows to generate heat as required.

Most of the advantages of this system are possible only when non-freezing solutions are used

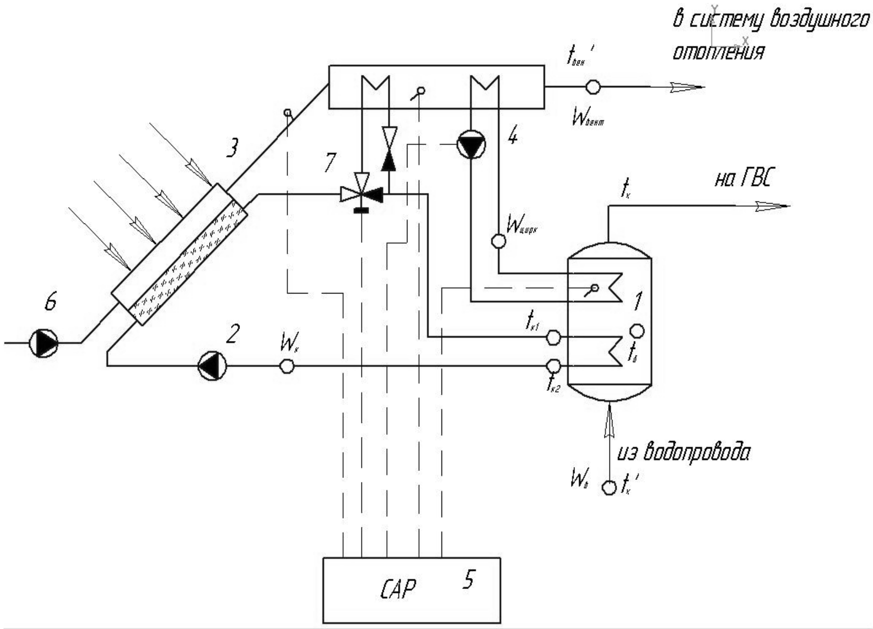

Fig. 1. Solar air heating system:
1 – storage tank; 2 – pump of solar contour; 3 – solar air collector; 4 - calorifier;
5 – automated control system; 6 – intake air ventilator; 7 – triple valve of the solar contour

in solar collectors. Air can be heated both in an intermediate heat exchanger from liquid heated in the collector, and directly in it. The Fig. 1 shows the scheme of the solar heat use allowing to combine these methods [3].

The given scheme will make it possible to use the advantages of air heating systems in the implementation of solar collectors of combined type (with combined heating of air and liquid heat-carrying medium) and to lower down the temperature of radiant heat absorbing plate, and so radiation loss as well.

The principle of the suggested scheme is as follows: a solar collector is attached through heated medium is to an air duct and antifreeze circulating in the contour «tank – accumulator – solar collector». Air through solar radiation equipment and due to forced convection in flowing tubes and a plate is heated, after which it is supplied to the air heating system. A storage tank through A storage tank through accumulating heat-carrying medium is connected to cold water pipe and extraneous heat source powered through submerged heating surface. At Fig. 1 connection circuit and operation mode of the solar plant are presented.

The collector efficiency (Fig. 1) can be considerably increased through intensification of heat exchange on surfaces with semispherical holes due to increased heat exchange surface area.

As early as in 1990-ies R.A. Serebryakov, G.I. Kiknadze, Yu.K. Krasnov and others in their works [4. 5, 6] studied a new class of quasipotential gas and liquid swirl flows formed either due to merging of specifically directed jets of working continuous medium, or due to streamlining its three-dimensional «holes» (so-called vortex generators) at heat exchange or carrying areas.

Highly-efficient practical application of such flows became possible due to large-scale thermodynamic research and design and development works. Thus, in accordance with experimental data, vortex flows formed in keeping with exact solutions of hydrodynamic equations increase gas or liquid consumption twice in comparison with flows with other structures at similar gas load in pipelines of similar size and form. Such swirled flows are self-organized in flows of energy carriers streamlining energy exchange surfaces formed by special profiles «vortex generator» [5, 6], considerably intensify heat and mass exchange while aerodynamic drag of energy exchange channels is reduced.

Calculation of daily heat absorption

To determine intensification efficiency, daily heat absorption of flat-plate water-air collector with hole intensification and without it [7].

Climatological data: Chita: longitude - 113 hours 23 min., latitude - 52 hours 6 min., October 1, 2013.

Surface orientation: incidence angle to horizon - 45 °, azimuth direction 180°.

Plate: accepted size 1000×100×1mm, material - copper, wall thickness - 2 mm.

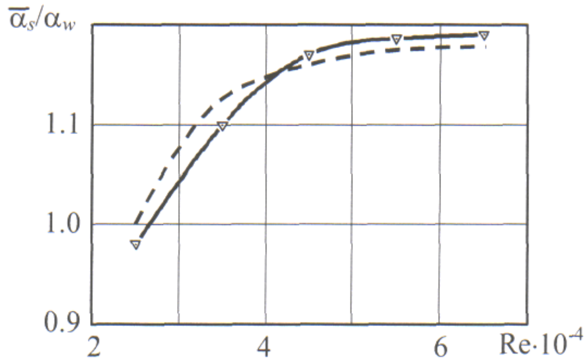

Fig. 2. Average (on the hole surface) relative coefficients of heat transfer for spherical hole

Pipes: number - 10, diameter - 25 mm, material - copper, wall thickness - 1 mm, coverage - 50%, thermal interface - tin, layer thickness - 500 μ.

Absorber: paint - mat black (degree of absorption - 95, blackness - 5, selectivity - 0.4).

External insulation: glass - two layers, thickness 1 mm, attenuation coefficient 0.4 1/mm, air passage size 30 mm.

Air parameters: air temperature at the inlet – 0 ^0C, air consumption - 1000 m^3/hour.

When calculating daily heat absorption of the regenerative water-air heat exchanger with high efficiency of heat exchange with the use of standard method, coefficients of heat transfer are increased in proportion to average (on the hole surface) relative coefficients of heat transfer. To consider increase of heat exchange surface, it is necessary to assess its relative increase K_F and include it in standard technique.

For suggested geometric parameters of hemispherical holes K_F assumes values from 1.09 to 1.116 in dependence on packing density (for preliminary calculation $K_F = 1{,}1$ was assumed). Calculations showed that flow pattern was turbulent: Re = 696364. For such pattern during intensification in spherical intensifier it is assumed that $\bar{\alpha} = 1.25$ (Fig. 2 [8]).

Results and their discussion

The results of the calculation of daily heat absorption of the flat-plate water-air collector are presented in Table 1.

Intensification provided efficiency increase by 2.43%. Yearly efficiency of the collector efficiency will amount to 106 kW/m^2.

Considering the current tariff in the Chita district, heat supply system (1 925 rouble/Gcal) economic efficiency is 175.5 rouble/m^2.

Table 1

Results of the calculation of a flat-plate collector
(air inlet temperature = ambient temperature = 0 °C)

Local time	Heat usefully absorbed by the collector (without intensification), w	Heat usefully absorbed by the collector (intensification: $K_F = 1.1, \bar{\alpha} = 1.25$), w
9:00	0	0
10:00	150	152
11:00	362	377
12:00	550	590
13:00	625	635
14:00	670	675
15:00	655	660
16:00	585	595
17:00	455	465
18:00	260	267
19:00	50	52
20:00	0	0
Total	4362	4468

Conclusions

The calculation of economic efficiency of the suggested scheme of solar air heating and intensification of the integrated collector by holes demonstrated considerable economic effect from the implementation of these methods.

References

1. Batukhtin A.G. Sovremennye metody povyshenija jeffektivnosti sovmestnoj raboty ustanovok geliootoplenija i sistem centralizovannogo teplosnabzhenija [Modern methods of increase of efficiency of collaboration of installations of helioheating and systems of the centralized heat supply] / A.G. Batukhtin, S.G. Batukhtin // Scientific and technical sheets of St. Petersburg politekhnichesy university. – 2009. – No. 3. – Page 48-53.
2. Al'ternativa kotel'nym est'! Otoplenie teplym vozduhom [The alternative to boiler rooms is! Heating by warm air] // Energy saving and problems of power industry of the Western Urals. – 2008. – No. 1-2. (July) – Page 18-22.
3. Patent Russian Federation No. 2403511. Solnechnaja ustanovka i sposob ee raboty [Solar installation and way of its work] / Batukhtin of A.G., Batukhtin S.G.// Bulletin of inventions. 2010. No. 31.
4. Kiknadze G. I. Samoorganizacija smercheobraznyh vihrevyh struktur v potokah gazov i zhidkostej i intensifikacija teplo- i massobmena [Self-organization the smercheobraznykh of vortex structures in streams of gases and liquids and an intensification warm and mass exchange] / G. I. Kiknadze, V. G. Olennikov//the Pre-print of institute of thermophysics of the Siberian Office of Academy of Sciences of the USSR. – Novosibirsk, 1990. - No. 227-90. – 46 pages.
5. The copyright certificate No. 247798, registration in State. Register of inventions of the USSR 04.01.1987 / Serebryakov R. A., Kiknadze G. I., Oxen V. T., Yudenkov N. A.
6. Patent Russian Federation No. 2546340. Kombinirovannyj solnechnyj vodovozdushnyj kollektor [The combined solar air-and-water collector] / Serebryakov R. A., Batukhtin S.G.// Bulletin of inventions. 2015. No. 10.
7. Bass M. S., Batukhtin A.G., Batukhtin S.G. Programma opredelenija optimal'nyh tehniko-jekonomicheskih pokazatelej raboty TJeS [Program of definition of optimum technical and economic indicators of work thermal power plant] / Certificate on the state registration of the computer program No. 2009614238.
8. Sapozhnikov S.Z. Osnovy gradientnoj teplometrii [Bases of a gradient teplometriya] / S.Z, Sapozhnikov, V. Yu. Mityakov, A.V. Mityakov. – S-Pb.: Publishing house of Polytechnical Universitea, 2012. - 203 pages.

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РЕГЕНЕРАТИВНЫЙ ВОДОВОЗДУШНЫЙ ТЕПЛООБМЕННИК

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Рассмотрена энергоэффективная схема использования энергии Солнца в системах централизованного теплоснабжения, позволяющая уменьшить капитальные затраты и эксплуатационные расходы и увеличить степень замещения традиционных источников теплоты. Универсальность систем с использованием данной схемы обусловливает широкую сферу их применения: от дома коттеджного типа до отопления промышленных помещений и тепличных комплексов. Сущность предлагаемой схемы заключается в том, что солнечный коллектор по нагреваемой среде подключают к воздуховоду, соединённому с отапливаемым помещением, и антифризом, циркулирующим в контуре: бак – аккумулятор – солнечный коллектор. Предложен способ интенсификации теплообмена за счет использования интенсификаторов луночного типа. Определена эффективность интенсификации посредством расчета суточного тепловосприятия плоского воздушного коллектора с интенсификацией лунками и без, а также экономи-

ческий эффект от внедрения предложенного способа. Согласно расчетам предложенная схема отопления и интенсификация теплообмена на поверхности входящего в неё коллектора могут дать ощутимый экономический эффект при внедрении.

Ключевые слова: солнечный коллектор, теплоноситель, конструкция, система моделирования, эксперимент, потребитель, регулирование, тепловая энергия, энергосбережение, нагрузка, оптимизация.

Литература

1. *Батухтин А.Г., Батухтин С.Г.* Современные методы повышения эффективности совместной работы установок гелиоотопления и систем централизованного теплоснабжения // Научно-технические ведомости СПбГПУ. – 2009. – №3. – С. 48-53.
2. Альтернатива котельным есть! Отопление теплым воздухом // Энергосбережение и проблемы энергетики Западного Урала. – 2008. – № 1-2 (июль) – С. 18-22.
3. Патент РФ № 2403511. Солнечная установка и способ ее работы / Батухтин А.Г., Батухтин С.Г. // БИ. 2010. № 31.
4. *Кикнадзе Г.И., Оленников В.Г.* Самоорганизация смерчеобразных вихревых структур в потоках газов и жидкостей и интенсификация тепло- и массобмена. Новосибирск: Институт теплофизики СО АН СССР, 1990. - №227-90. - 46 с.
5. А.С. №247798, регистрация в Гос. Реестре изобретений СССР 04.01.1987 г./ Серебряков Р.А., Кикнадзе Г.И., Волов В.Т., Юденков Н.А.
6. Патент РФ №2546340. Комбинированный солнечный водовоздушный коллектор / Серебряков Р.А., Батухтин С.Г. // БИ. 2015. №10.
7. *Басс М.С., Батухтин А.Г., Батухтин С.Г.* Программа определения оптимальных технико-экономических показателей работы ТЭС: Свидетельство о государственной регистрации программы для ЭВМ №2009614238.
8. *Сапожников С.З., Митяков В.Ю., Митяков А.В.* Основы градиентной теплометрии. С.-Пб.: Изд-во Политехнического Университета, 2012. – 203 с.

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ACCELERATORS OF LOW-POTENTIAL WIND FLOW IN WIND POWER PLANTS

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The article presents the prospects of the use of wind power plants for energy supply to relatively small-size and distributed objects situated in zones with low load density. The necessity of researching the application of various design of flow accelerators of various design to increase wind power plants efficiency in regions with low wind activities is revealed.

Keywords: *wind receiving device, flow concentrator, confuser, diffuser, Venturi tube, efficiency, regions with low wind activities, wind flow accelerators.*

Introduction

For more economical energy supply to consumers it is necessary to use various energy sources, both centralized and local. Therefore, recently in Russia and abroad research for the use of renewable energy sources has been expanded. The use of power plants converting wind and water energy into any other energy is primarily designed to improve energy supply to relatively small and distributed objects located in remote regions with low load density situated far from large-scale electric grids and oil and gas pipelines [1].

The analysis of existing wind power plants demonstrates that they are effectively used in regions with annual average wind velocity above 7 m/s where daily and monthly wind velocity histograms are even. It should be mentioned that wind power plants with small blades have proved themselves most effective in the above mentioned regions. However, conducted research shows that in regions with annual average wind velocity of 4-7 m/s high-speed small-blade wind power plants (specific speed Z = 6–9) operate in design mode from 152 to 720 hours, or 2-8 % per year [2].

The Analysis of Existing Models of Wind Flow Accelerators

Basic requirements made by consumers and experts to wind power plants are aimed at provision of steady operation of power plants even at low

Fig. 1. Flow concentrators designed to increase efficiency of wind energy use

wind velocity (3-3,5 м/с). Currently most relevant is research of the use of flow accelerators of various design to raise efficiency of wind power plants in regions with low wind activities. Recently a whole range of proposals have been developed for the application of additional devices (flow concentrators, flow accelerating elements) designed to raise efficiency of the wind energy use, in power plants design (Fig. 1) [3].

Common peculiarity of these power plants is the use of various types of flow-directing devices or flow concentrators, for organized input and output of air flow to the working wheel. Flow concentrators are confuser of diffuser devices installed in close vicinity to the working wheel of an energy plant. It is assumed that in the result of their action the velocity of flow in the wheel zone and consequently, flow energy efficiency are increased.

However, the problem of concentrating air flow proved to be not quite simple. Even such simple devices as confusers proved to be inefficient. If the ratio of the diameters of inlet and outlet holes is insignificant and amounts to 1.3-1.5, flow velocity can be increased by 20–25%. Further increase of the confuser inlet hole does not result in increased velocity. But even such increase will raise the wind power plant efficiency twice. But it should be understood that similar energy increment can be obtained by increasing the swept area of wind power plant twice. In this case the size of the wind power plant blades should be increased 1.4 times (up to the size of the confuser inlet hole). It turns out that the use of confusers is inefficient – it is easier to enlarge the blades size than to build a structure with a confuser that in addition must be wind- oriented.

Simulation of operation of tandem boosters of wind flow

The theory of a perfect wind power plant theory limiting coefficient of wind energy usage (CWEU) by the value of 59.3%, assumes that air outcoming from the wind energy plant with velocity three times less than wind velocity, extends to infinity as solid cylinder. In practice, even at insignificant distance from the wind energy plant encircling flow erodes outgoing cylinder and accelerates it. Acceleration reduces pressure which is transmitted to the back side of the blades thus providing additional capacity. That is why concentrators directing jets of external rapid air inside outcoming slow flow, result in increased CWEU [4].

In accordance with the classical theory of an ideal wind power plant, the loss of velocity in the wind wheel plane is equal to one third of wind velocity, and total wind velocity loss behind the wind wheel exceeds the velocity loss in the rotation plane twice. Therefore, wind velocity in the plane of the WPP is arithmetic average of wind velocity in front of the wind wheel and wind velocity behind it

$$V_{BK} = \frac{V_1 + V_2}{2}, \quad (1)$$

where V_1 – wind velocity in front of the wind wheel; V_2 – wind velocity after the wind wheel.

Let us replace the wind wheel by circular air permeable disc (Fig. 2) and assume that at the disc section so much energy is taken from wind that as velocity behind the disc only reaches the value $V_2 = 1/3V_1$ (in ideal case).

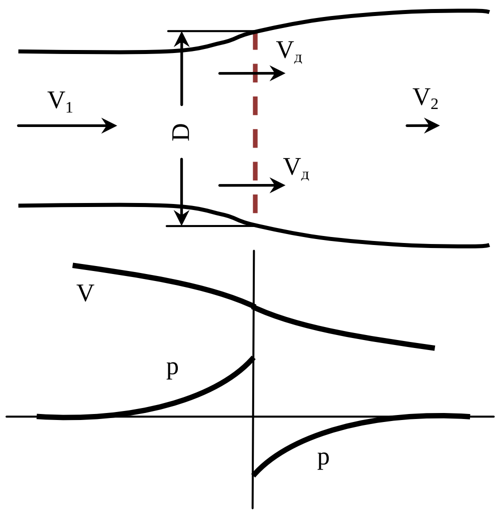


Fig. 2. Schematic representation of the air-permeable disc operation

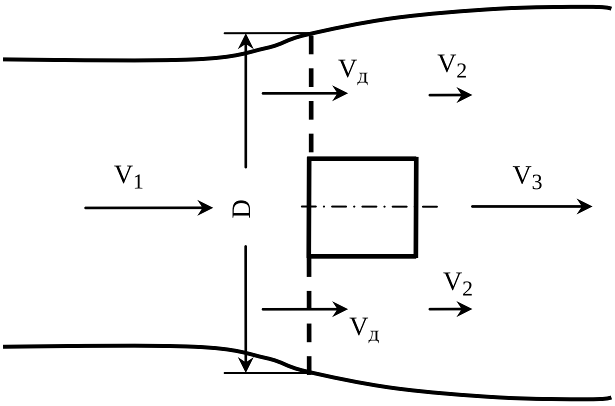
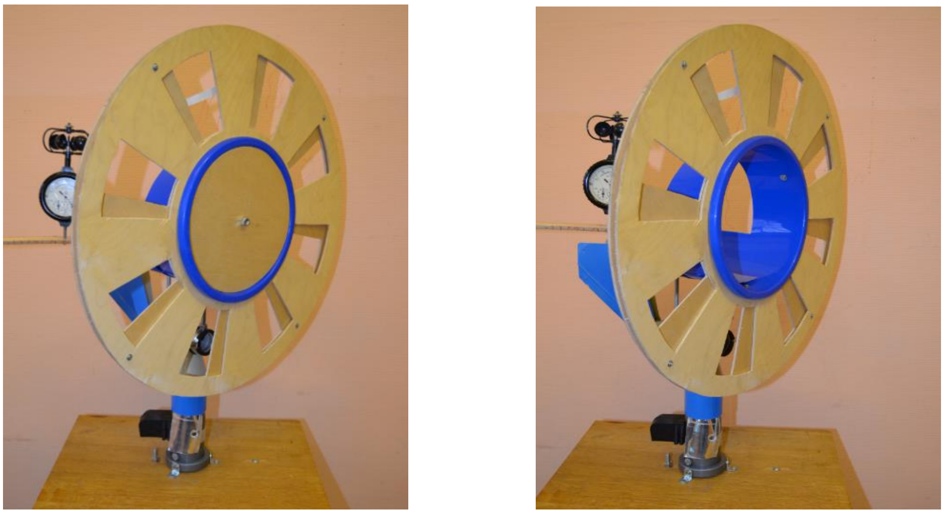


Fig. 3. Schematic representation of operation of the disc equipped with the tube



a **b**

Fig. 4. Testing bench for simulating the disc operation in the «no-tube» mode (a) and in the mode with the tube (b)

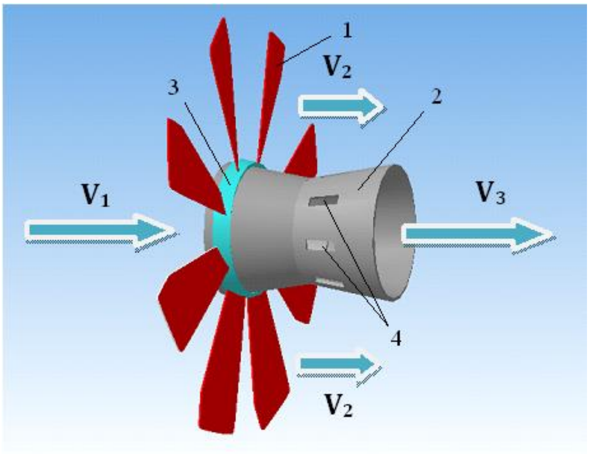
This phenomenon results in slight air retention (deceleration) by the wind wheel. Its velocity is transformed into pressure as in the case when a body is falling onto spring platform where kinetic energy is transformed into spring tension. Consequently, air is supplied to the wheel with reduced velocity but with increased pressure. To generate higher energy at the wind wheel it is first of all necessary to reduce kinetic pressure behind the wind wheel by increasing velocity.

It is known that the internal part of the wind wheel of all conventional impeller wind power plants remains unused because of structural peculiarities and low efficiency (wind flow in front of the wind wheel is crushed, incidence angle is changed and consequently, efficiency is reduced). That is why it is assumed that one third of actual radius of the wind wheel is not used [5].

In accordance with the formula (1), in order to increase velocity in the disc $V_д$, section, it is necessary to increase wind velocity behind the disc V_2. For this purpose at the center of the disc the tube with diameter d = D/3 is installed and the disc is blown over by wind flow with the same velocity V_1 (Fig. 3).

In this case the tube will not impede easy passage of wind flow, so at the outlet the velocity of the flow V_3 will be little less than V_1. Therefore, in air flow behind the disc velocity gradient will be observed and due to viscosity (particles coalescence with each other) air layers moving faster, carry away layers moving slower, velocity behind the disc V_2 is increased and, consequently, velocity in section of the disc $V_д$ too.

Fig. 4 presents the testing bench for simulation of the wind-receiving device operation. The testing bench is a disc through which air flow is



a

b

Fig. 5. Wind receiving device with the wind flow accelerator:
a - computer model; b - physical model

passed through special slots with variable size. While the slots size is increased or decreased, the drag coefficient of the wind receiving device is also changed.

In the process of experiment, the disc is swept over by wind flow in the «no-tube» mode (the central part with diameter d = D/3 is closed) and in the mode with the tube (the central part is opened)/ With the use of the anemometer the indicators of wind velocity behind the disc V_2 at different drag coefficients are taken. The experiment demonstrated that in the mode with the tube velocity V_2 is increased in comparison with the «no-tube» operation mode at the same drag coefficient.

One of the aims of our research is the development of the design of the wind-receiving device with a wind flow accelerator [6, 7]. Fig. 5 (a, b) presents computer and physical models of the wind-receiving device. The wind-receiving device comprises the wind wheel *1* and the aerodynamic flow accelerator made in the form of the Venturi tube *2* and located in the center of the wind-receiving device. The wind wheel *1* is freely rotating at the bearing *3* installed at the Venturi tube *2*, being the wind wheel axis.

In addition, the wind receiving device is equipped with the special holes 4 through which rapid air flow (driving flow) passing via the Venturi tube carries away slow external air flow (driven flow) thus generating some rareficition behind the wind wheel *1* which in its turn generates rarefication behind the wind wheel and increases the velocity of air flow passing through the wind wheel.

The use of the suggested wind-receiving device with a tandem aerodynamic booster of wind flow will make it possible to increase both daily and annual energy output and will raise installed capacity utilization factor up to 52%.

Conclusions

Basic competitive advantages of wind energy plants with wind flow accelerators are as follows: annual energy output increase up to 400 % depending on annual average wind velocity; increase of installed capacity utilization factor up to 55–62 %; minimal service maintenance. Moreover, the use of composite materials will not only make cheaper the structure and alleviate it, but will also increase the wind power plants service life.

Wind energy plants with wind flow accelerators can be applied for electric power supply to consumers distributed at the territory with low specific load (agricultural consumers, farms, fisheries, hunting entities, individual farms, etc.), as well as for electrification of social infrastructure of territories (cellular communications, information support, meteorological stations, EMERCOM posts, video surveillance, security functions, monitoring, etc.).

Serial production of small-size wind power plants will allow to implement effective power supply systems in decentralized regions and will make the idea of using wind energy more popular among the population and administrations of RF regions.

References

1. Shefter Ya.I. Use of wind power. Moscow, Energoatomizdat, 1983.
2. Andrianov V.N., Bystritsky D.N., Vashkevich of K.P. Wind and power plants. Moscow, State power publishing house, 1960.
3. Alternative power engineering in the world today and forecasts for tomorrow. - Access mode: http://vce-znau .ru/fizika/5840/index.html? page=2
4. Rozin M. N. The concentrators accelerating the departing stream. - Access mode: http://www.rosinmn.ru/
5. Betts A. Wind power and its use by means of wind engines. Kharkov, State scientific and technical publishing house of Ukraine, 1933.
6. Patent No. 142342. The wind turbine with the active accelerator of a wind stream / Dorzhiyev S.S., Bazarova E.G., Gorinov of K.A. Date of the publication: 22.05.2014.
7. Patent No. 143120. The wind turbine with the passive accelerator of a wind stream / Dorzhiyev S.S., Bazarova E.G., Gorinov K.A. Date of the publication: 10.06.2014.

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УСКОРИТЕЛИ НИЗКОПОТЕНЦИАЛЬНОГО ВЕТРОВОГО ПОТОКА В ВЕТРОУСТАНОВКАХ

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В статье рассмотрена перспектива использования ветроустановок для энергоснабжения относительно небольших и рассредоточенных объектов, расположенных в зонах с малой плотностью нагрузки. Выявлена необходимость исследования по применению различных конструкций ускорителей потока для повышения эффективности ветровых энергоустановок применительно к районам низкой ветровой активности.

Ключевые слова: *ветроприемное устройство, концентратор потока, конфузор, диффузор, труба Вентури, эффективность, район низкой ветровой активности, ускоритель ветрового потока.*

Литература

1. *Шефтер Я.И.* Использование энергии ветра. М.: Энергоатомиздат, 1983.
2. *Андрианов В.Н., Быстрицкий Д.Н., Вашкевич К.П.* Ветроэлектрические станции. М.: Государственное энергетическое издательство, 1960.
3. Альтернативная энергетика в мире сегодня и прогнозы на завтра: Электронный ресурс. Режим доступа: http://vce-znau.ru/fizika/5840/ index.html?page=2
4. *Розин М.Н.* Концентраторы, ускоряющие отходящий поток. – Режим доступа: http://www.rosinmn.ru/
5. *Бетц А.* Энергия ветра и ее использование посредством ветряных двигателей. Харьков: Государственное научно-техническое издательство Украины, 1933.
6. Патент РФ №142342. Ветроустановка с активным ускорителем ветрового потока / Доржиев С.С., Базарова Е.Г., Горинов К.А. Опубликовано: 22.05.2014.
7. Патент РФ №143120. Ветроустановка с пассивным ускорителем ветрового потока / Доржиев С.С., Базарова Е.Г., Горинов К.А. Опубликовано: 10.06.2014.

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THE LIANIT THEORY OF CYCLOTOMIC EQUATIONS

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An efficient algorithm based on lianit algebra has been developed for the calculation of nth primitive roots to cyclotomic equations. The developed approach and its applications have a number of advantages over the standard Galois theory.

Keywords*: cyclotomic equations, lianit roots, transforming polynomials, nth primitive roots, dummy index identities*

Introduction

The n^{th} cyclotomic equation (CE) for any positive integer n is defined [1], [2] as a polynomial equation, whose roots are the n^{th} roots of unity,

$$C^n(x) = x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \cdots + x + 1) = 0.$$

Since Gauss [3], the only theoretical method of calculating the n^{th} primitive roots of unity has been the so called method of Gaussian periods. In essence, Gauss had developed a specific modification of Galois theory for $f^n(x) = x^n - 1$. However, in [4], [5] CE of $n = 5, 7$ were solved via the lianit solutions [6], [7] of pseudo-algebraic polynomials [8], [9] without group-theoretical reasonings or that of Gaussian periods of classical cyclotomy. In [9] the 11^{th} roots of unity were calculated using a similar approach. The famous case of Gauss's heptadecagon [3] was also solved in [9] and all primitive 16^{th} roots of unity were calculated in quadratic radicals. These results led us to conclude that the lianit theory is capable of producing a unified algebraic solution to the problem of CE. In this paper we formulate such an algorithm based on a two-element lianit algebra and provide a universal method for calculating the n^{th} roots of $x^n \pm 1 = 0$ for any natural n. The paper is organized as follows.

In Section 2 we prove a general theorem that establishes the standard form of the pseudo-polynomial $f^n(\sigma)$ if its numeric analogue shares common roots with a quadratic polynomial. This important theorem provides a deep insight into the possible structure of pseudo-polynomials made up of weighed superposition of various degrees of lianits. The existence of a two-parametric polynomial $\varphi^{n-1}(p,q)$ of degree $n-1$ entering the standard form of $f^n(\sigma)$ manifests the solvability of arbitrary degree polynomial in radicals. We conclude Section 2 by discussing two important consequences of the theorem proved: the first is a purely algebraic proof of the theorem of multiple roots of polynomials (without resorting to the concept of continuity). The second consequence establishes the conditions of "collinearity" between two pseudo-polynomials. The collinearity between two pseudo-polynomials is understood as proportionality relation between their respective elements. The latter property is interesting since lianits are departures from matrices and/or complex numbers. The property of "collinearity" between two polynomials as non-linear superposition of lianits might serve as basis for further research into building a possible lianit calculus. In Section 3 we apply the formulae received in Section 2 to develop a universal algorithm of solving CE of any natural degree. We discover a general parametrical polynomial $f^{\frac{n-1}{2}}(p)$ whose parametric solutions p_i generate exactly $\frac{n-1}{2}$ quadratic equations $x^2 + p_i x + 1 = 0$ sharing $n-1$ numeric roots with the required CE. We introduce the term *transforming polynomial (TP)* for $f^{\frac{n-1}{2}}(p)$. We show that the roots of TP satisfy certain *dummy index identities* $p_i p_j = -(p_v + p_w)$ $(i \neq j,\ v \neq w)$ that ensure the solvability of TP by radicals. We conclude Section 3 by providing a universal formulae for calculating the coefficients of transforming polynomials for any equation $x^n \pm 1 = 0$ with odd n. In Section 4 we provide a general criterion of solvability for any quantic equation and calculate by way of illustration the 5th primate roots of unity. In Section 5 we consider the equation of heptagon and calculate all the 7th primitive roots of unity. Section 6 illuminates in great detail each step of solving the equation of hendecagon so the reader may fully appreciate the universality and straightforwardness of the developed non-numeric theory of CE. All 11th primitive roots are calculated. In Section 7 we successfully extend our algorithm over the case of CE $x^n \pm 1 = 0$ with even n. Finally, the results of our work are discussed in the Summary.

Solvability by Radicals and Transforming Polynomials

In this paper we use the lianit algebra, well-known from our previous research [6], [7], [8], [9].

$$\sigma_1 \cdot \sigma_2 = (x_1, x_2)(y_1, y_2) = [x_1(y_1 + y_2), x_2 y_1] \tag{1}$$

with right-hand unity element $e = (1, 0)$ and $k = (k, 0)$ as the analogue of the complex number k.

Theorem. Let the polynomials $f^n(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$ and $f^2(x) = x^2 + {} +px + q$ share at least one common complex root x_{02}, and let $\sigma = (x_1, x_2)$ be the principal lianit root of $f^2(x)$ within the algebra (1). Then there exists a polynomial $\varphi^{n-1}(p, q)$ of degree $n - 1$ such that,

$$f^n(\sigma) = \left[x_{01} \cdot \varphi^{n-1}(p, q);\ \frac{q}{p}\ \varphi^{n-1}(p, q)\right]; (p \neq 0), \tag{2}$$

where x_{01} is the other complex root of $f^2(x)$.

Proof. By the Theorem's hypothesis $\sigma = (x_1, x_2)$ is the principal lianit root of $f^2(x) = x^2 + px + q$, thereby $\sigma = (x_1, x_2) = \left(-p, \frac{q}{p}\right)$ since within the set (1), the equation $f^2(\sigma) = \sigma^2 + \sigma p + q = (0, 0)$ leads to the only possible lianit root $\sigma = (x_1, x_2) = \left(-p, \frac{q}{p}\right)$. Consider σ^m, where m is arbitrary natural number and the polynomial $f^m(x) = x^m + ax + b$, for which the lianit $\sigma = (x_1, x_2)$ serves as a secondary root within the same set (1). In that case $\sigma_1 = (x_{01}, 0)$ and $\sigma_2 = (x_{02}, 0)$ are equivalent to the complex roots of $f^m(x) = x^m + ax + b$ (see the Theorem of Principal Lianit Roots [6], [7]) i.e., $f^m(\sigma) = \sigma^m + a\sigma + b = (0, 0)$. Hence,

$$\begin{cases} x_{01}^m + a x_{01} + b = 0, \\ x_{02}^m + a x_{02} + b = 0, \end{cases} \Rightarrow a = \frac{x_{02}^m - x_{01}^m}{x_{01} - x_{02}}\ ;\ \ b = x_{01}x_{02}\left(\frac{x_{01}^{m-1} - x_{02}^{m-1}}{x_{01} - x_{02}}\right). \tag{3}$$

From $f^m(\sigma) = \sigma^m + \sigma \cdot a + b = (0, 0)$ we have: $\sigma^m = (x_1, x_2)^m = (-a x_1, -a x_2) + (-b, 0) = (-a x_1 - b, -a x_2)$. Taking (3) into account we obtain,

$$\sigma^m = (x_1, x_2)^m = \left[x_1 \cdot \frac{x_{01}^m - x_{02}^m}{x_{01} - x_{02}} + x_{01}x_{02}\frac{x_{02}^{m-1} - x_{01}^{m-1}}{x_{01} - x_{02}}; x_2 \cdot \frac{x_{01}^m - x_{02}^m}{x_{01} - x_{02}}\right]. \tag{4}$$

To compile the lianit $f^n(\sigma) = \sigma^n + \sigma^{n-1} \cdot a_1 + \sigma^{n-2} \cdot a_2 + \cdots + \sigma \cdot a_{n-1} + a_n$ we employ the formula (4), at $m = n-1, n-2, \ldots$. The second element of $f^n(\sigma)$ is the sum,

$$x_2 \frac{x_{01}^n - x_{02}^n}{x_{01} - x_{02}} + a_1 x_2 \frac{x_{01}^{n-1} - x_{02}^{n-1}}{x_{01} - x_{02}} + \cdots + a_{n-2} x_2 \frac{x_{01}^2 - x_{02}^2}{x_{01} - x_{02}} + x_2 a_{n-1} =$$
$$= x_2 \left[\frac{(x_{01}^n - x_{02}^n) + a_1(x_{01}^{n-1} - x_{02}^{n-1}) + \cdots + a_{n-2}(x_{01}^2 - x_{02}^2)}{x_{01} - x_{02}} + a_{n-1} \right] = \tag{5}$$
$$= x_2 \varphi^{n-1}(x_{01}, x_{02}).$$

Given $\sigma = (x_1, x_2) = \left(-p, \frac{q}{p}\right)$, we rewrite (5) as: $x_2 \varphi^{n-1}(x_{01}, x_{02}) \equiv \frac{q}{p} \varphi^{n-1}(p, q)$. For the second element of $f^n(\sigma)$, using the Vieta relations $x_{01} + x_{02} = -p$, $x_{01} x_{02} = q$, the formula (4) gives,

$$x_1 \frac{x_{01}^n - x_{02}^n}{x_{01} - x_{02}} + x_{01} x_{02} \frac{x_{02}^{n-1} - x_{01}^{n-1}}{x_{01} - x_{02}} + a_1 x_1 \frac{x_{01}^{n-1} - x_{02}^{n-1}}{x_{01} - x_{02}} + a_1 x_{01} x_{02} \frac{x_{02}^{n-2} - x_{01}^{n-2}}{x_{01} - x_{02}} + \cdots$$
$$+ a_{n-2} x_1 (x_{01} - x_{02}) + a_{n-2} x_{01} x_{02} + a_{n-1} x_1 + a_n. \tag{6}$$

Since, by the Theorem's hypothesis, x_{02} is the common complex root of $f^2(x)$ and $f^n(x)$, the expression (6), being the first element of the lianit $f^n(\sigma) = \sigma^n + \sigma^{n-1} \cdot a_1 + \cdots + \sigma \cdot a_{n-1} + a_n$, may be readily written as,

$$x_{01} \left[\frac{(x_{01}^n - x_{02}^n) + a_1(x_{01}^{n-1} - x_{02}^{n-1}) + \cdots + a_{n-2}(x_{01}^2 - x_{02}^2)}{x_{01} - x_{02}} + a_{n-1} \right] = x_{01} \varphi^{n-1}(p, q) \tag{7}$$

(in view of the identity $x_{02}^n + a_1 x_{02}^{n-1} + a_2 x_{02}^{n-2} + \cdots + a_{n-1} x_{02} + a_n = 0$) which proves the theorem.

As a direct consequence of the proven Theorem, one can derive, strictly algebraicly, the conditions for the existence of multiple (double) root of the polynomial $f^n(x)$. Setting $x_{01} = x_{02}$ and removing the idle multiplier $x_{01} - x_{02}$, in (5) and (6) beforehand, we derive the expressions: $(n-1)x_{02}^{n-2} + (n-2)x_{02}^{n-3} + \cdots + a_{n-1}$ and $(x_{02}^n + a_1 x_{02}^{n-1} + \cdots + a_{n-1} x_{02} + a_n) + +[(n-1)x_{02}^{n-2} + (n-2)x_{02}^{n-3} + \cdots + a_{n-1}]$. On the other hand, the condition $x_{01} = x_{02}$ implies that the lianit $\sigma = (x_1, x_2) = \left(-p, \frac{q}{p}\right) = \left(2x_{01}, -\frac{x_{01}}{2}\right) = \left(2x_{02}, -\frac{x_{02}}{2}\right)$ renders into a secondary lianit root for $f^n(\sigma)$, that is,

$$f^n(\sigma) = \left\{ f^n(x_{02}) + x_{02}[f^n(x_{02})]' \,;\, -\frac{x_{02}}{2}[f^n(x_{02})]' \right\} = (0, 0),$$

where by $[f^n(x_{02})]' = (n-1)x_{02}^{n-2} + (n-2)x_{02}^{n-3} + \cdots + a_{n-1}$ we have denoted the first derivative of the polynomial $f^n(x)$, at the point $x_{02} = x_{01}$. The set of equations $f^n(\sigma) = (0, 0)$, obviously stipulates that the values of $f^n(x)$ and its derivative at $x_{01} = x_{02}$ simultaneously vanish. From (2) follows another consequence. Let $\sigma = (x_1, x_2)$ be the principal lianit root of $f^2(x)$ with numeric roots x_{01}, x_{02}

and let $f^m(x)$, $f^\ell(x)$ are any given polynomials with at least one common root with $f^2(x)$ (which we take as x_{02}). Then the corresponding elements of $f^m(\sigma)$ and $f^\ell(\sigma)$ are proportional.

Indeed, according to the Theorem above within the set (1) for both $f^m(x)$ and $f^\ell(x)$ there exist polynomials $\varphi_1^{m-1}(p,q)$, $\varphi_1^{\ell-1}(p,q)$ such that,

$$f^m(\sigma) = \left[x_{01}\varphi_1^{m-1}(p,q), \frac{q}{p}\varphi_1^{m-1}(p,q)\right] = (y_1, y_2),$$

$$f^\ell(\sigma) = \left[x_{01}\varphi_2^{\ell-1}(p,q), \frac{q}{p}\ \varphi_2^{\ell-1}(p,q)\right] = (z_1, z_2).$$

Hence it readily follows, $\frac{y_1}{z_1} = \frac{y_2}{z_2} = \frac{\varphi_1^{m-1}(p,q)}{\varphi_2^{\ell-1}(p,q)}$. It is not hard to see, that this property remains valid in any N-element lianit sets with commutative rule of addition and distributive rule of multiplication. If $f^n(x)$ holds a principal lianit root $\sigma = (x_1, x_2, \dots, x_N)$ and its $n-1$ complex roots out of n are common with $f^m(x)$ and $f^\ell(x)$ then, the elements of lianits $f^m(\sigma) = (y, y_2, \dots, y_N)$ and $f^\ell(\sigma) = (z_1, z_2, \dots, z_N)$ satisfy the condition: $\frac{y_i}{z_i} = const$ $(i = 1, 2, \dots, N)$, for $N \geq n$.

The Lianit Algorithm for the n^{th} Roots of Unity

We now investigate the general problem of finding the n^{th} roots of $x^n - 1 = 0$ via the formula (2) from the already proved Theorem. To establish explicitly the function $\varphi_1^{n-1}(p,q)$ one needs to perform an identity comparison between the elements of the lianits $f^n(\sigma) = \sigma^n + \sigma^{n-1}\cdot a_1 + \cdots + \sigma \cdot a_{n-1} + a_n$ and (2). It is easy to see, that within the set (1) for the first element of the lianit $\sigma^n = \sigma\cdot\sigma\cdots(\sigma\cdot 1) = (x_1, x_2)^n$ we have,

$$\eta_1^n = x_1^n + \frac{(n-1)}{1!}x_1^{n-1}\cdot x_2 + \frac{(n-2)(n-3)}{2!}x_1^{n-2}x_2^2 + \frac{(n-3)(n-4)(n-5)}{3!}x_1^{n-3}x_2 + \frac{(n-4)(n-5)(n-6)(n-7)}{4!}x_1^{n-4}x_2^4 + \frac{(n-5)(n-6)(n-7)(n-8)(n-9)}{5!}x_1^{n-5}x_2^5 + \cdots \tag{8}$$

The number of summands in (8) is exactly $\frac{n+1}{2}$, if n is odd and $\frac{n+2}{2}$ if n is even. For the second element of the lianit $\sigma^n = (x_1, x_2)^n$, following the multiplication algorithm in (1), we have, $\eta_2^n = = x_2\cdot\eta_1^{n-1}$, where η_1^{n-1} is the first element of σ^{n-1}. The sum (8) can be written more compactly as,

$$\eta_1^n = \sum_{i=0}^{\frac{n-1}{2}} C_i^{n-i}x_1^{n-i}\cdot x_2^i\,, if\ n = 2k+1; \qquad \eta_1^n = \sum_{i=0}^{\frac{n}{2}} C_i^{n-i}x_1^{n-i}\cdot x_2^i\,, if\ n = 2k. \tag{9}$$

Let $f^n(\sigma) = \sigma^n\cdot 1 + \sigma^{n-1}\cdot a_1 + \cdots + \sigma\cdot a_{n-1} + a_n = [z_1(x_1,x_2), z_2(x_1,x_2)]$. Comparing it with the lianit (2) and taking account the Vieta relations $x_{01} + x_{02} = -p$ we obtain,

$$x_{02} = -\left[\frac{p \cdot \varphi^{n-1}(p,q) + z_1(p,q)}{\varphi^{n-1}(p,q)}\right]. \tag{10}$$

According to the hypothesis of the theorem, x_{02} is the common numeric root between $f^2(x) = x^2 + px + q$ and $f^n(x) = x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$. Substituting x_{02} from (10) into the equation $f^2(x) = x^2 + px + q = 0$ leads to a general parametrical equation $F^n(p,q) = 0$ of degree n with respect to p or q. For each fixed value of the parameter q, with respect to p we obtain an equation $F_0^n(p,q_0) = 0$ the numeric roots p_i $(i = 1, 2, \dots, n)$ of which have the property: at least one of the numeric roots of the equation $x^2 + p_ix + q_0 = 0$ is also root to $f^n(x) = x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$. In case of $f^n(x) = x^n - 1$, the comparision of $f^n(\sigma) = \sigma^n - 1$ with (2.2) yields,

$$\sigma^n - 1 = (\eta_1^n - 1, \eta_2^n) = (\eta_1^n - 1, x_2 \cdot \eta_1^{n-1}) = \left[x_{01}\varphi^{n-1}(p,q), \frac{q}{p}\varphi^{n-1}(p,q)\right]. \tag{11}$$

In view of $x_{01} + x_{02} = -p$, we obtain,

$$x_{02} = -\left[\frac{p \cdot \eta_1^{n-1} + \eta_1^n - 1}{\eta_1^{n-1}}\right]; \;\; \eta_1^{n-1} \equiv \varphi^{n-1}(p,q). \tag{12}$$

Substituting x_{02} from (2.12) into the $x^2 + px + q = 0$ leads to a general parametric equation of degree n with respect to p (q appears as a free parameter). Since one of the numeric roots of $f^n(x) = x^n - 1$ is the unity, whatever the chosen value of $q = q_0$, the restraint $1 + p_0 \cdot 1 + q_0 = 0$ lowers the degree of the parametric equation $F^n(p,q) = 0$ by one: $F^n(p,q) = [p + (1 + {} +q_0)]F_0^{n-1}(p,q_0)$. From the general formula (2) the condition $\varphi^{n-1}(p,q) = 0$ turns automatically the lianit $\sigma = (x_1, x_2)$ into secondary lianit root for $f^n(x)$ as $f^n(\sigma) = (0,0)$. Analyzing $\varphi^{n-1}(p,q)$ through x_{01}, x_{02} using (7), and taking, that $F_0^{n-1}(p,q) = 0$, we see, that at $q_0 = 1 = x_{01} \cdot x_{02}$ the polynomials $\varphi^{n-1}(p,1) = \eta_1^{n-1}(p,1)$ and $F_0^{n-1}(p,1)$ have common numeric roots, that is, the polynomials thereof have a common form-factor (of parameter p) of degree $\ell = \frac{n-1}{2}$. Therefore the set of primitive n[th] roots of unity can be found as roots of quadratic equations $x^2 + p_ix + 1 = 0$, where p_i are the numeric roots of the abovesaid polynomials of degree $\frac{n-1}{2}$. To standardize the developed formalism we introduce for these polynomials the term *transforming polynomial* (*TP)* for the cyclotomic (and related palindromic) equations taken in the form: $x^n \pm 1 = 0$. Thus, within the framework of the suggested theory, the problem of searching for the solutions of CE renders to the problem of calculating their corresponding TPs.

1. Based on the results above we are now able to formulate an algorithm for finding the TP for any CE $x^n - 1 = 0$, where n is odd natural number:

a) The TP is of degree $\ell = \frac{n-1}{2}$ and its roots satisfy the dummy index identities $p_ip_j = -(p_v + p_w)$ $(i \neq j, v \neq w)$.

b) The absolute values of the coefficients $\delta_1, \delta_2, \dots, \delta_\ell$ of the TP $f^\ell(p) = f^{\frac{n-1}{2}}(p) = p^\ell + +\delta_1 p^{\ell-1} + \delta_2 p^{\ell-2} + \cdots + \delta_\ell$ are determined by the rule,

$$\delta_k = \frac{\left(\ell - \frac{k}{2}\right)!}{(\ell-k)!\left(\frac{k}{2}\right)!} \qquad if\ k\ is\ even, \tag{13}$$

$$\delta_k = \frac{\left(\ell - \frac{k+1}{2}\right)!}{(\ell-k)!\left(\frac{k-1}{2}\right)!} \quad if\ k\ is\ odd. \tag{14}$$

c) The sign of the coefficients δ_k $(k = 1, 2, \dots, \ell = \frac{n-1}{2})$ are determined by signature $(-,-,+,+,-,-,+,+,\dots)$.

2. For the equation $x^n + 1 = 0$ (n is again an odd number) the coefficients of the TP are given by the same formulas (13)-(14), but the signs δ_k are given by signature $(+,-,-,+,+,-,-,\dots)$ since the roots of the TP differ from the roots of $f^{\frac{n-1}{2}}(p)$ only by sign. The identity for the roots p_i in this case is: $p_i p_j = (p_v + p_w)$; $i \neq j$, $v = w$. The dummy index identity $p_i p_j = -(p_v + p_w)$ for numeric roots p_i of TP $f^{\frac{n-1}{2}}(p) = f^\ell(p)$ obviously means, that in a decomposition $f^{\frac{n-1}{2}}(p) = (p^2 + a_1 p + a_2) \cdot \left(p^{\frac{n-5}{2}} + b_1 p^{\frac{n-7}{2}} + \cdots + b_{\frac{n-5}{2}}\right)$ the possible coefficients a_1, a_2 are the roots of one and the same equation of the order $N = \frac{\left(\frac{n-1}{2}\right)!}{2!\left(\frac{n-5}{2}\right)!}$, where n is the degree of the CE $x^n - 1 = 0$.

The index identity holding for any TP $f^{\frac{n-1}{2}}(p)$ definitely guarantees the solvability of the corresponding equations $f^{\frac{n-1}{2}}(p) = 0$ in radicals. The calculation of parameters p_i $(i = 1, 2, \dots, \frac{n-1}{2})$ allows to find the primitive roots of n^{th} CE $f^n(x) = x^n \pm 1$ for odd values of n. If n is a prime number and $\frac{n-1}{2}$ has dividers $5, 7, 11, \dots$, the roots of TP are readily calculated via the Lagrange resolvents in conjunction with the index identity restraints: $p_i p_j = -(p_v + p_w)$. This is the case, for example with $x^{11} - 1 = 0$. If, on the contrary, n is a prime number and $\ell = \frac{n-1}{2}$, then the index identity relations trivially simplify the root finding procedure (this is the case when $n = 5, 7, 13, 17, 19, \dots$). Of course, of particular interest are the well-known classical cases of odd prime n, namely: $n = 11, 13, 17, 19$. The corresponding TPs are readily calculated via the algorithms (13)-(14), wherein, for any n, the general parametric condition $F^n(p,q) = 0$, at $q = 1$ assumes: $F^n(p,1) = (p+2) \cdot \left[f^{\frac{n-1}{2}}(p)\right]^2$, where $f^{\frac{n-1}{2}}(p)$ is the TP for $x^n - 1 = 0$ (the case with $p = -2$, $q = 1$, trivially maps to $x^2 - 2x + 1 = (x-1)^2 = 0$ since one of the roots of $x^n - 1 = 0$ is always the unity). To help our reader get a hands-on experience with the developed lianit formalism of cyclotomic equations, we turn now to several important illustrations.

Illustration 1: The Quintic Equation

Consider the parametric conditions for the quintic polynomial taken in the normal form: $f^5(x) = x^5 + a_0x + b_0$. Comparing the lianit $f^5(\sigma) = \sigma^5 + a_0\sigma + b_0$ with (2) we have,

$$\varphi^4(p,q) = p^4 - 3p^2q + q^2 + a_0; \quad x_{02} = \frac{-p^3q + 2pq^2 - b_0}{p^4 - 3p^2q + q^2 + a_0}. \tag{15}$$

Substituting x_{02} into $x^2 + px + q = 0$ we arrive at a parametric equation,

$$b_0p^5 - a_0qp^4 - 5b_0qp^3 + 4a_0q^2p^2 + (a_0b_0 + 5b_0q^2)p - (q^5 + 2a_0q^3 + a_0^2q + b_0^2) = 0. \tag{16}$$

Any pair (p,q) satisfying (16) defines uniquely a trinomial $f^2(x) = x^2 + px + q$ one of the roots of which is a root for the original $f^5(x) = x^5 + a_0x + b_0$. If one takes $a_0 = 0$, $b_0 = -1$, we have the cyclotomic case $f^5(x) = x^5 - 1$, and taking in (16) the free parameter as $q = 1$, we obtain, as expected: $-p^5 + 5p^3 - 5p - 2 = -(p+2)(p^2 - p - 1)^2$, where $f^2(p) = p^2 - p - 1$ is the TP for $x^5 - 1 = 0$. As in the case of $n = 11$, the nominator and the denominator in the fraction (15) are divisible by $f^2(p) = p^2 - p - 1$, that is: $-p^3 + 2p + 1 = -(p+1)(p^2 - p - 1)$; $p^4 - 3p^2 + 1 = (p^2 + p - 1)(p^2 - p - 1)$. For $f^2(p) = p^2 - p - 1$ the identity of indices checks out: $p_1 \cdot p_2 = -(p_1 + p_2) = 1$. Since $n = 5$ is a Fermat number, the primitive roots of $f^5(x) = x^5 - 1$ can be expressed in square radicals. This indeed, is the case: $p^2 - p - 1 = 0$; $p_{1,2} = \frac{1 \pm \sqrt{5}}{2}$. The 5th primitive roots are thus found by solving the quadratic equations: $x^2 + \left(\frac{1 \pm \sqrt{5}}{2}\right)x + 1 = 0$.

Illustration 2: The Equation of Heptagon

For $f^7(x) = x^7 - 1$, from (13)-(14), we readily obtain the TP: $f^3(p) = p^3 - p^2 - 2p + 1$. All six primitive roots coincide with the roots of $x^2 + p_ix + 1 = 0$, where p_1, p_2, p_3 are the roots of $p^3 - p^2 - 2p + 1 = 0$. The index identities for p_1, p_2, p_3 are: $p_1p_2 = -(p_2 + p_3)$; $p_1p_3 = -(p_1 + p_2)$; $p_2p_3 = -(p_1 + p_3)$, in full accordance with the Vieta relations for the cubicals $f^3(p)$.

Illustration 3: The Equation of Hendecagon

For the so called Vandermonde's case $f^n(x) = x^{11} - 1$ we raise the required lianit $\sigma = (x_1, x_2)$ to degree $n = 11$ by (8) and comparing $\sigma^{11} - 1 = (x_1, x_2)^{11} - (1,0)$ with (2), we have,

$$\varphi^{10}(p,q) = p^{10} - qp^8q + 28p^6q^2 - 35p^4q^3 + 15p^2q^4 - q^5, \tag{17}$$

$$x_{02} = \frac{-p^9q + 8p^7q^2 - 25p^5q^3 + 20p^3q^4 - 5pq^5 + 1}{p^{10} - 9p^8q + 28p^6q^2 - 35p^4q^3 + 15p^2q^4 - q^5} \tag{18}$$

Substituting x_{02} into $f^2(x) = x^2 + px + q$, we obtain the general parametric condition for the $f^2(x) = x^2 + px + q$ and $f^{11}(x) = x^{11} - 1$ to have at least one common numeric root x_{02}. That is,

$$p^{11} - 11p^9q + 44p^7q^2 - 77p^5q^3 + 55p^3q^4 - 11pq^5 + q^{11} + 1 = 0. \quad (19)$$

For any $q = q_0$ the equation (19) can be represented as $[p + (1 + q_0)]F^{10}(p, q_0) = 0$. For each of the ten roots of the equation $F^{10}(p, q_0)$ =0 there is a single quadratic equation $x^2 + p_i x +$ $+q_0 = 0$, one of the roots of which is the primitive root to $f^{11}(x) = x^{11} - 1$. Choosing $q_0 = -1$, we have: $p(p^{10} + 11p^8 + 44p^6 + 77p^4 + 55p^2 + 11)$ to illustrate how the degree of the general TP can be reduced by one. At $q = 1$ the denominator of $[\varphi^{10}(p, q)]$ and (19) have a common multiplier in form of a polynomial of degree $\frac{n-1}{2} = 5$, that is, the TP reads,

$$f^5(p) = p^5 - p^4 - 4p^3 + 3p^2 + 3p - 1. \quad (20)$$

in exact accordance with the derived algorithm of finding the TP by (13)-(14). All ten primitive roots of $x^{11} - 1 = 0$ with the roots of the set of quadratic trinomials $x^2 + p_i x + 1 = 0$ $(i = 1, 2, 3, 4, 5)$, in other words $\sigma_i = (x_1^i, x_2^i) = \left(-p_i, \frac{1}{p_i}\right)$ $(i = 1, 2, \dots, 5)$ being principal roots to $f^2(x) = x^2 + p_i x + 1$ are secondary lianit roots for $f^{11}(x) = x^{11} - 1$.

As was stated above, the case $n = 11$ utilizes Lagrange resolvents. The decomposition of (20) by the scheme $f^5(p) = (p^2 + a_1 p + a_2)(p^3 + b_1 p^2 + b_2 p + b_3)$ is ineffective. This is resulted by the existence of index identities $p_i p_j = -(p_v + p_w)$; $(i \neq j,\ v \neq w)$, which stipulate the unknown coefficients a_1, a_2 to be the roots of the one and the same equation of the 10th order $(\frac{5!}{2!\cdot 3!} = 10)$, namely,

$$a_{1,2}^{10} + 4a_{1,2}^9 - 6a_{1,2}^8 - 35a_{1,2}^7 - 8a_{1,2}^6 + 67a_{1,2}^5 + 37a_{1,2}^4 - 28a_{1,2}^3 - 13a_{1,2}^2 + 3a_{1,2} + 1 = 0. \quad (21)$$

Thus in (21) the set of values for $a_1 = -(p_i + p_j)$ simply coincides with $a_2 = p_v p_w$ implying that (21) cannot have factorizing polynomials with integer coefficients lower the 5th degree. In fact, only from five expressions of the structure $-(p_i + p_j)$ $(i \neq j)$ one may receive a sum with an integer value, for instance, take the set, $-(p_1 + p_5), -(p_2 + p_3), -(p_2 + p_4), -(p_1 + p_4), -(p_3 + p_5)$ with the sum $-2(p_1 + p_2 + p_3 + p_4 + p_5) = -2$. This evidently infers, that (21) may be decomposed only into factoring polynomials of the 5th degree: $f^5(a_1) = f^5(a_2) = a_{1,2}^5 + k_1 a_{1,2}^4 +$ $+k_2 a_{1,2}^3 + k_3 a_{1,2}^2 + k_4 a_{1,2} + k_5$, with k_i as the conjectured integers. One can easily verify, that (21) decomposes as follows,

$$\left(a_{1,2}^5 + 2a_{1,2}^4 - 5a_{1,2}^3 - 2a_{1,2}^2 + 4a_{1,2} - 1\right)\left(a_{1,2}^5 + 2a_{1,2}^4 - 5a_{1,2}^3 - 13a_{1,2}^2 - 7a_{1,2} - 1\right) = 0.$$

The identity of indices in combination with Lagrange resolvents for the polynomial $f^5(p) = p^5 - p^4 - 4p^3 + 3p^2 + 3p - 1$ effectively achieves the goal. The system of Lagrange resolvents for $f^5(p)$ is,

$$\begin{cases} 5p_1 - 1 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4, \\ 5p_2 - 1 = A\alpha_1 + D\alpha_2 + C\alpha_3 + B\alpha_4, \\ 5p_3 - 1 = B\alpha_1 + C\alpha_2 + D\alpha_3 + A\alpha_4, \\ 5p_4 - 1 = C\alpha_1 + A\alpha_2 + B\alpha_3 + D\alpha_4, \\ 5p_5 - 1 = D\alpha_1 + B\alpha_2 + A\alpha_3 + C\alpha_4. \end{cases} \tag{22}$$

In (22) A, B, C, D are the primitive roots to $x^5 - 1 = 0$ which being the solutions to $x^2 + \frac{1 \pm \sqrt{5}}{2}x + 1 = 0$ read,

$$A = \frac{-1 - \sqrt{5} - \sqrt{-10 + 2\sqrt{5}}}{4}; \quad B = \frac{-1 - \sqrt{5} + \sqrt{-10 + 2\sqrt{5}}}{4};$$
$$C = \frac{-1 + \sqrt{5} - \sqrt{-10 - 2\sqrt{5}}}{4}; \quad D = \frac{-1 + \sqrt{5} + \sqrt{-10 - 2\sqrt{5}}}{4}. \tag{23}$$

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ – are the Lagrange resolvents,

$$\alpha_1 = p_1 + Dp_4 + Cp_5 + Bp_2 + Ap_3; \quad \alpha_2 = p_1 + Dp_3 + Cp_2 + Bp_4 + Ap_5;$$
$$\alpha_3 = p_1 + Dp_2 + Cp_3 + Bp_5 + Ap_4; \quad \alpha_4 = p_1 + Dp_5 + Cp_4 + Bp_3 + Ap_2. \tag{24}$$

Raising the equations of the system (22) to degrees $2, 3, 4, 5$, and taking into account the Vieta relations for the $f^5(p) = p^5 - p^4 - 4p^3 + 3p^2 + 3p - 1$, we have,

$$\begin{cases} \alpha_1\alpha_4 + \alpha_2\alpha_3 - 22 = 0; \qquad \alpha_1\alpha_4 = \alpha_2\alpha_3 \\ \alpha_1^2\alpha_3 + \alpha_2^2\alpha_1 + \alpha_3^2\alpha_4 + \alpha_4^2\alpha_2 + 11 = 0, \\ \alpha_1^3\alpha_2 + \alpha_2^3\alpha_4 + \alpha_3^3\alpha_1 + \alpha_4^3\alpha_3 + 11 \cdot 31 = 0, \\ \alpha_1^5 + \alpha_2^5 + \alpha_3^5 + \alpha_4^5 - 11 \cdot 89 = 0. \end{cases} \tag{25}$$

In (25) the additional condition $\alpha_1\alpha_4 = \alpha_2\,\alpha_3 = 11$ follows from the identity $p_ip_j = -(p_v + p_w)$. Indeed from (24) we have,

$$\alpha_1\alpha_4 = (p_1^2 + \cdots p_5^2) + (C + D)(p_1p_5 + p_2p_3 + p_2p_4 + p_1p_4 + p_3p_5)$$
$$+ (A + B)(p_1p_2 + p_1p_3 + p_4p_5 + p_3p_4 + p_2p_5),$$
$$\alpha_2\alpha_3 = (p_1^2 + \cdots p_5^2) + (C + D)(p_1p_2 + p_1p_3 + p_4p_5 + p_3p_4 + p_2p_5)$$
$$+ (A + B)(p_1p_5 + p_2p_3 + p_2p_4 + p_1p_4 + p_3p_5).$$

As the roots p_i and p_j $(i \neq j)$ enter each of the brackets symmetrically, with the help of the index identities, we receive: $p_1p_5 + p_2p_3 + p_2p_4 + p_1p_4 + p_3p_5 = p_1p_2 + p_1p_3 + p_4p_5 + p_3p_4 + {}$ $+p_2p_5 = -2(p_1 + p_2 + p_3 + p_4 + p_5) = -2$. Since $p_1^2 + p_2^2 + p_3^2 + p_4^2 + p_5^2 = 9$ and $C + D = \frac{-1+\sqrt{5}}{2}$, $A + B = \frac{-1-\sqrt{5}}{2}$, then $\alpha_1\alpha_4 = \alpha_2\alpha_3 = 11$.

Introducing new denotions $y=\alpha_1^3\alpha_2$, $z=\alpha_1^2\alpha_3$ and taking into account that $\alpha_2\alpha_3=11$ we have $y\cdot z=11\alpha_1^5$. Then the second and the third equations of the system (25) can be rewritten as,

$$\begin{cases} y^2+\left(\dfrac{z^2+11z+11^3}{11}\right)\cdot y+11z^2=0, \\ y^2+\left(\dfrac{31\cdot 11\cdot z^2}{z^2+11^3}\right)y+11z^2=0. \end{cases} \tag{26}$$

As in the case with (25), the equations of (26) are identical at $\frac{z^2+11z+11^3}{11}=\frac{31\cdot 11\cdot z^2}{z^2+11^3}$, that is,

$$z^4+11z^3-9\cdot 11^2z^2+11^4z+11^6=0. \tag{27}$$

which in its turn decomposes into a product of two quadratic trinomials,

$$\left[z^2+\frac{11}{2}\left(1+5\sqrt{5}\right)z+11^3\right]\left[z^2+\frac{11}{2}\left(1-5\sqrt{5}\right)z+11^3\right]=0. \tag{28}$$

Let's take, for instance, one of the roots of the trinomial from the first bracket, (28). We have,

$$z=\frac{11}{4}\left[\left(-1-5\sqrt{5}\right)+i\frac{5\left(1+5\sqrt{5}\right)}{\sqrt{5+2\sqrt{5}}}\right] \tag{29}$$

the corresponding value of y from (26), calculates,

$$y=\frac{11}{4}\left[\left(-31-5\sqrt{5}\right)-i\frac{5\left(1+3\sqrt{5}\right)}{\sqrt{5+2\sqrt{5}}}\right];\ \left(i=\sqrt{-1}\right). \tag{30}$$

Taking that $y\cdot z=11\alpha_1^5$, we obtain the value of α_1 as,

$$\alpha_1=\left[\frac{11}{4}\left(89+25\sqrt{5}\right)+i\frac{5\left(5-7\sqrt{5}\right)}{\sqrt{5+2\sqrt{5}}}\right]^{\frac{1}{5}}. \tag{31}$$

hence we have finally:

$$5p_1-1=\alpha_1+\alpha_2+\alpha_3+\alpha_4=\alpha_1+\frac{y}{\alpha_1^3}+\frac{z}{\alpha_1^2}+\frac{11}{\alpha_1}. \tag{32}$$

The exact values for the primitive roots of $x^{11}-1=0$ are recevied as roots to the quadratic equations: $x^2+p_ix+1=0$, $(i=1,2,\dots,5)$. The equation with integer coefficients for α_1^5 reads,

$$\alpha_1^{20}-11\cdot 89\alpha_1^{15}+11^3\cdot 35\alpha_1^{10}-11^6\cdot 89\alpha_1^5+11^{10}=$$
$$=\left[\alpha_1^{10}-\frac{11}{2}\left(89+25\sqrt{5}\right)\alpha_1^5+11^5\right]\left[\alpha_1^{10}-\frac{11}{2}\left(89-25\sqrt{5}\right)\alpha_1^5+11^5\right].$$

For reference purposes, we write out the system of dummy index identities $p_ip_j=-(p_v+p_w)$ for the TP $f^5(p)=p^5-p^4-4p^3+3p^2+3p-1$ in the expanded form,

$$\begin{aligned} &p_1p_4=-(p_2+p_5), \quad p_3p_4=-(p_2+p_4), \\ &p_2p_3=-(p_4+p_5), \quad p_1p_3=-(p_3+p_5), \\ &p_2p_4=-(p_1+p_3), \quad p_1p_2=-(p_1+p_4), \\ &p_1p_5=-(p_3+p_4), \quad p_4p_5=-(p_1+p_5), \\ &p_3p_5=-(p_1+p_2), \quad p_2p_5=-(p_2+p_3). \end{aligned} \tag{33}$$

Cyclotomic Equations of even n

Since the theorem from Section 2 is valid for arbitrary polynomials $f^n(x)$, it also gives us the algorithm for construing TPs for $f^n(x) = x^n \pm 1$, where n is any even number. Of course, the non-trivial cases are $n = 8, 16, 32, \dots$. Using the same reasoning as in Section 3, the algorithm of constructing TP for any $f^n(x) = x^n - 1$, formulates as:

a) The TP has degree of $N = \frac{n}{2}$, i.e. $f^{\frac{n}{2}}(p)$, one of its roots is always equal to 2 as the number $x_{01} = -1$ is a solution to $x^n - 1$ (as well as $x_{02} = +1$).

b) The coefficients of the TP $f^{\frac{n}{2}}(p) = f^{\ell}(p) = p^{\ell} + \delta_1 p^{\ell-1} + \delta_2 p^{\ell-2} + \cdots + \delta_{\ell}$ are determined by the rule,

$$If\ k\ is\ an\ even\ index, then \qquad \delta_k = \frac{\left(\ell - 1 - \frac{k}{2}\right)!}{(\ell - 1 - k)!\left(\frac{k}{2}\right)!} \tag{34}$$

$$If\ k\ is\ an\ odd\ index, then \quad \delta_k = \frac{\left(\ell - \frac{k+1}{2}\right)!}{(\ell - k)!\left(\frac{k-1}{2}\right)!} \tag{35}$$

c) The sequence of signs of the coefficients are given by the signature: $(-, -, +, +, -, -, +, +, \dots)$.

The secondary lianit roots of $x^n - 1 = 0$ within the set (1), for any even n, are given by $\sigma_i = \left(-p_i, \frac{1}{p_i}\right)$, where p_i are the numeric roots of $f^{\frac{n}{2}}(p)$. The numeric roots of $x^n - 1 = 0$ are the roots of the quadratic equations: $x^2 + p_i x + 1 = 0$; $(i = 1, 2, \dots, \frac{n}{2})$. It is readily appreciated that the algorithm of finding TP for $f^n(x) = x^n + 1$ (n is even), follows quite straightforwardly, though, obviously, they also appear in the TPs of $f^{2n}(x) = x^{2n} - 1 = 0$ as form-factors of their corresponding $f^n(p)$. The TP for $x^n \pm 1 = 0$ for concrete cases $n = 8, 16, 32, \dots$ may be found in our next publications.

Summary

We solved the general n[th] degree cyclotomic equation using a purely non-numeric algebraic approach. The developed formalism does not use any group-theoretical reasonings or the Gaussian periods of classical cyclotomy. On the other hand, the lianit-algebraic approach has computational and methodological advantages over the Galois Group Theory: it is straightforward and methodologically simpler. It does not require construction of specific algebraic structures but rather all the information on the roots is already "packed" in the original lianit algebra. In this sense, the lianit-algebraic approach is better suited to solve the cyclotomic equations. Complete computation of the n[th] roots up to n=32 will be provided in our next paper.

References

1. S. Lang, *Algebra*, Springer. pp. 276–277, (2002).
2. B. L. van der Waerden, *Algebra*, **1**, Springer, (1967).
3. Gauss, Carl F., *Disquisitiones Arithmeticae*, Yale University Press, pp. §§359–360, (1965).
4. L.V. Hakobyan, *NAS RA*, **108**, 2, ISSN 0321-1339, (2008).
5. Akopyan L.V., Akopyan V.S., *Math. Reports NAS RA*, **110**, 4, 348-358, (2010).
6. L. V. Akopyan, *Proceedings of the Yerevan State University*, **2**, 23-34 (2007).
7. L. V. Akopyan, *Proceedings of the Yerevan State University*, **3**, 33-43 (2007).
8. L. V. Akopyan, *Proceedings of the Yerevan State University*, **4**, 21-35 (2007).
9. L. V. Akopyan, *State Engineering University of Armenia*, "Non-numeric Roots of Pseudo-algebraic Equations", Ph.D. Dissertation, (2008).

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ЛИАНИТОВАЯ ТЕОРИЯ УРАВНЕНИЙ ДЕЛЕНИЯ КРУГА

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Разработан новый теоретический аппарат для расчета первообразных (примитивных) корней степени n из единицы (ПКЕ) в основном поле на основе лианитовой теории псевдоалгебраических уравнений. В основе предлагаемой методики расчета ПКЕ положена простая модель двухэлементной лианитовой алгебры, хорошо знакомая из предыдущих работ. Предварительно доказывается оригинальная теорема о специальном виде псевдомногочленов $f^n(\sigma)$, чьи числовые аналоги имеют хотя бы один общий комплексный корень с квадратными уравнениями. В рамках этой теоремы показывается существование и единственность специального двухпараметрического числового полинома $\varphi^{n-1}(p,q)$ степени $n-1$, однозначно определяющего вид псевдомногочлена $f^n(\sigma)$. Появление $\varphi^{n-1}(p,q)$ фактически является частным критерием разрешимости полиномов в радикалах. Приложение данной теоремы к классу так называемых уравнений деления круга (УДК) приводит к общему параметрическому многочлену $f^{\frac{n-1}{2}}(p)$ степени $\frac{n-1}{2}$, параметрические решения p_i которого порождают ровно $\frac{n-1}{2}$ квадратных уравнений $x^2+p_ix+1=0$, обладающих $n-1$ общими корнями с исходным УДК. Для многочлена $f^{\frac{n-1}{2}}(p)$ вводится новый термин – *переходной многочлен (ПМ)*. Показано, что корни ПМ удовлетворяют вполне определенному *тождеству условных индексов* $p_ip_j=-(p_v+p_w)$ $(i\neq j,\ v\neq w)$, смысл которых кроется в разрешимости ПМ в радикалах. Тем самым вычисление ПКЕ сводится к определению ПМ. Выведена общая формула для расчета коэффициентов ПМ для произвольных УДК $x^n\pm1=0$ с нечетным n. На основе разработанного теоретического аппарата предложен общий алгоритм для практических расчетов первообразных корней произвольных степеней. Разработанная теория затем переносится и на случай УДК с четным n. Предлагаемая методика иллюстрирована на примерах вычисления ПКЕ для $n=5,7,11$.

Ключевые слова: уравнения деления круга, первообразные корни из единицы, псевдоалгебраические уравнения, лианитовые корни, переходной многочлен, тождества условных индексов.

Литература

1. *Ленг С.* Алгебра. М.: Мир, 1968.
2. *Б. Л. Ван дер Варден.* Алгебра. М.: Наука, 1979.
3. *Гаусс К.Ф.* Арифметические исследования / Пер. с нем. – М.: АН СССР, 1959.

4. *Акопян Л.В.* Об уравнении деления круга // Доклады Академии Наук РА. 2008, вып. 108, ч. **2**, с. 133-141.
5. *Акопян Л.В., Акопян В.С.* Решение уравнения деления круга без периодов Гаусса // Доклады Академии Наук РА. 2010, вып. **110**, ч. 4, с. 348-358.
6. *Акопян Л.В.* Нечисловые корни алгебраических уравнений: сообщение I // Ученые записки Ереванского Государственного Университета. 2007. № **2**, с. 23-34.
7. *Акопян Л. В.* Нечисловые корни алгебраических уравнений: сообщение II // Ученые записки Ереванского Государственного Университета. 2007, №**3**, с. 33-43.
8. *Акопян Л.В.* // Ученые записки Ереванского Государственного Университет. 2007, №**4**, с. 21-35.
9. *Акопян Л.В.* Нечисловые корни псевдоалгебраических уравнений: дис. … канд. физ.-мат. наук, А.01.06: «Алгебра и теория чисел». Ереван: Государственный инженерный университет Армении, 2008.

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