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AN ALGEBRAIC PROOF TO THE FUNDAMENTAL THEOREM OF ALGEBRA

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The Fundamental Theorem of Algebra is proved utilizing the concepts and methods of non-numeric roots of algebraic polynomials.

Keywords: *principal and secondary lianit roots, pseudo-algebraic equations, associated polynomial, numeric roots, fundamental theorem of algebra.*

Introduction

The existence and the properties of specific non-numeric roots to algebraic equations were introduced in works [1], [2]. These non-numeric roots were named *lianit* roots. Lianit roots were first introduced as extension of complex roots of algebraic equations. The specific properties of lianit roots and their relationship to the existent complex roots of algebraic equations were further clarified in [3]. The lianits were defined as arbitrary tables of a finite number of complex numbers within a given algebra of addition and multiplication. Lianits turned out to be generalizations for both complex numbers and matrices. The rules of composition in lianit algebras were allowed to violate commutability, associativity and distribuitivity. The term lianit was originally coined in view of the fact that such algebras allowed to render the problem of finding the explicit expressions for numeric roots of algebraic equations to systems of linear equations.

In further works a number of applications of lianit algebras were discovered. In [4], [5] the lianits were shown to be effective at solving a system of two polynomial equations with two variables at arbitrary degrees. The condition of existence of a k-fold numeric root for a given polynomial was obtained via the lianit algebras without resorting to the methods of mathematical analysis. The explicit expressions for the numeric roots of polynomial equations up to the fourth degree were derived different in form from those of Cardano's and Ferrari's. Classes of solvable quintics were isolated and several conditions of sovability in radicals were received similar to those of Galois conditions [6]. Vieta's relations for the numeric roots were successfully generalized over the linanit roots.

In [7], [8] lianit algebras allowed to obtain all possible complex roots of Cyclotomic Equations [9] in the general case without resorting to the Gaussian periods [10]. In the Dissertation Paper [11] the obtained results were systematized. To distinguish between usual algebraic equations with complex numeric roots and those where the variable is treated as a lianit the term *pseudo-algebraic equation* was introduced.

The aim of this work is to demonstrate how the Fundamental Theorem of Alegbra (FTA) can be proved using purely algebraic concepts and methods within the framework of lianit algebras [1], [11]. Two such proofs are presented using two specially constructed lianit algebras.

The paper is organized as follows. In Section 2 we review the main concepts, definitions and theorems of lianit algebras drawing upon [1], [2], [11]. We also derive some of the formulae necessary for the proof of the FTA later in the paper.

In Section 3 we provide a proof of the FTA using a specific lianit algebra and the properties of secondary lianit roots of pseudo-algebraic equations. In Section 4 we provide another proof of FTA by using a different lianit algebra and broaden the idea of *associated polynomial* introduced in Section 2. The results of our work are discussed in the Summary. Finally, some of the calculative aspects of our work are gathered in the Appendix.

Review of Lianit Algebras

A table of N arbitrarily arranged complex numbers is called an N-element lianit. According to the definitions given in [1], [2], [11] an N-element lianit algebra Σ is the set of lianits $\sigma = (x_1, x_2, \dots x_l \dots, x_N)$, $\tau = (y_1, y_2, \dots y_l \dots, y_N)$, $\dots, \omega = (z_1, z_2, \dots z_l \dots, z_N) \dots$, where x_ℓ, y_ℓ, $\dots$ ($\ell = 1, 2, \dots, N$) are complex numbers, with the following rules:

1. The sum $\sigma + \tau$ and the product $\sigma \cdot \tau$ are also N-element lianits belonging to Σ.

2. For an arbitrary complex number k there exists in Σ a right-handed lianit $\kappa = (\varepsilon_1(k), \varepsilon_2(k), \ldots, \varepsilon_N(k))$. We will frequently refer to κ as the lianit analog of the complex number k. The functions $\varepsilon_1(k), \varepsilon_2(k), \ldots, \varepsilon_N(k)$ are some universal complex variable functions for a given algebra.

3. There exists trivial element $\theta = [\varepsilon_1(0), \varepsilon_2(0), \ldots, \varepsilon_N(0)]$ such that $\sigma \cdot \theta = (x_1, x_2, \ldots, x_N) \cdot (\varepsilon_1(0), \varepsilon_2(0), \ldots, \varepsilon_N(0)) =$ $= (0,0,\ldots,0)$.

The requirments $1-3$ provide some general confinements over the complex functions $\varepsilon_1(k), \varepsilon_2(k), \ldots, \varepsilon_N(k)]$. From $1-3$ it also follows that a lianit algebra may or may not have a right identity element. In the general case such an element $e = (\varepsilon_1(1), \varepsilon_2(1), \ldots, \varepsilon_N(1))$ exists if the lianit analog κ of the complex number k possesses the property

$$\sigma \cdot \kappa = (x_1, x_2, \ldots, x_N) \cdot [\varepsilon_1(k), \varepsilon_2(k), \ldots, \varepsilon_N(k)] =$$
$$= (kx_1, kx_2, \ldots, kx_N),$$

which is yet another limittion over the complex variable functions $\varepsilon_1(k), \varepsilon_2(k), \ldots, \varepsilon_N(k)]$.

In the general case the properties of commutability, associativity and distributivity with respect to addition and multiplication of elements are not held for lianit algebras [1], [2], [3]. In the case of the right-handed lianit analog of complex number k the product $\sigma_1 \cdot \sigma_2 \cdots \sigma_n \cdot \kappa$ is composed strictly from left to right,beginning with the last right couple $\sigma_n \cdot \kappa$ i.e. $\sigma_1 \cdot [\sigma_2 \cdots (\sigma_n \cdot \kappa)]$. In the same fashion, if the addition is not commutative, one should define the sequence of summation in advance.

Consider the polynomial $f^n(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$ with a single complex variable x and complex coefficients a_i, $i = 1,2,\ldots,n$. If we present the summand $a_i x^{n-i}$ as an N-element lianit from a prescribed algebra Σ, i.e. $a_i x^{n-i} \Rightarrow \sigma \cdot [\sigma \cdots (\sigma \cdot a_i)]$, then the resulting $f^n(\sigma) = \sigma^n \cdot 1 + \sigma^{n-1} \cdot a_1 + \cdots + \sigma \cdot a_{n-1} + a_n$ will be an N-element lianit within the same algebra. Strictly speaking we ought to call this new polynomial pseudo-algebraic following [11]. However, we will habitually drop this term in order not to complicate the text and in further both the $f^n(x)$ and its lianit analog $f^n(\sigma)$ will be called simply polynomials.

Definition. *The lianit* $\sigma = (x_1, x_2, \ldots, x_N)$ *is called a lianit root to the* $f^n(\sigma)$ *within a prescibed algebra* Σ *if* $f^n(\sigma) = \theta = [\varepsilon_1(0), \varepsilon_2(0), \ldots, \varepsilon_N(0)]$.

Definition. *The lianit* $\sigma = (x_1, x_2, \ldots, x_N)$ *is called the principal lianit root of* $f^n(\sigma)$, *if* $f^n(\sigma)$ *is the only polynomial of* n*-th degree for which* $\sigma = (x_1, x_2, \ldots, x_N)$ *is a root. Otherwise the lianit* $\sigma = (x_1, x_2, \ldots, x_N)$ *is called a secondary root for* $f^n(\sigma)$. *It is quite simple to see that if* $N < n$, *the condition* $f^n(\sigma) = (0,0,\ldots,0)$ *stipulates* $\sigma = (x_1, x_2, \ldots, x_N)$ *to be a secondary root [1].*

Consider the set of n-element ($N = n$) lianits within the framework of the following algebra

$$\begin{cases} \sigma_1 + \sigma_2 = (x_1, x_2, \ldots, x_n) + (y_1, y_2, \ldots, y_n) = \\ \qquad = (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n) = \sigma_2 + \sigma_1 \\ \sigma_1 \cdot \sigma_2 = (x_1, x_2, \ldots, x_n) \cdot (y_1, y_2, \ldots, y_n) = \\ \qquad = [x_1(y_1 + y_2), x_2(y_1 + y_3), \ldots, x_{n-1}(y_1 + y_n), x_n y_1] \neq \sigma_2 \sigma_1. \end{cases} \tag{1}$$

Algebra (1) is not commutative and associative but is distributive with respect to multiplication:

$$\sigma_1(\sigma_2 + \cdots + \sigma_n) = \sigma_1\sigma_2 + \cdots + \sigma_1\sigma_n.$$

Choose the lianit analog to the complex number k as $\kappa = (k,0,\ldots,0)$. The right identity is $e = (1,0,\ldots,0)$.Any n_0-th degree polynomial (1„ n_0„ n) with non-trivial coefficients has a single principal root in (1)

$$f^n(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n;\ \sigma_1 = \left(-a_1, \frac{a_2}{a_1}, \frac{a_3}{a_2}, \ldots, \frac{a_n}{a_{n-1}}\right);\ a_i \neq 0$$
$$f^{n-1}(x) = x^{n-1} + b_1 x^{n-2} + \cdots + b_{n-2}x + b_{n-1}; \sigma_2 = \left(-b_1, \frac{b_2}{b_1}, \frac{b_3}{b_2}, \ldots, \frac{b_{n-1}}{b_{n-2}}, 0\right);$$
$$b_i \neq 0 \tag{2}$$
$$f^{n-2}(x) = x^{n-2} + c_1 x^{n-3} + \cdots + c_{n-3}x + c_{n-2}; \sigma_3 = \left(-c_1, \frac{c_2}{c_1}, \frac{c_3}{c_2}, \ldots, \frac{c_{n-2}}{c_{n-3}}, 0, 0\right);$$
$$c_i \neq 0$$
$$\ldots\ \ldots\ \ldots$$
$$f^1(x) = x + a_0;\ \sigma_0 = (-a_0, 0, \ldots, 0);\ a_0 \neq 0.$$

However, within the same algebra (1) had we chosen the lianit analog to the complex number as $\kappa = (0,0,\ldots,0,k)$ only $f^n(x)$ would have had a principal root $\sigma_1 = \left(-a_1, \frac{a_2}{a_1}, \frac{a_3}{a_2}, \ldots, \frac{a_n}{a_{n-1}}\right)$, secondary roots do not exist. In case of $k = (k,0,\ldots,0)$, secondary roots exist for all polynomials of degrees $1 < n_0,, n$, along with the principal roots (2)).

The theorem of principal lianit roots formulates [1]:

Suppose the lianit $\sigma = (x_1, x_2, \ldots, x_n)$ *is the principal root to the polynomial* $f^n(x)$ *within an* n*-element algebra with commutative summation and distributive multiplication laws and suppose* σ *is a secondary root to some other polynomial* $f^m(x) = x^m + b_1 x^{m-1} + \cdots + b_{m-1}x + b_m$ *(*$m > n$*):* $f^n(\sigma) = (0,0,\ldots,0)$, $f^m(\sigma) = (0,0,\ldots,0)$. *Then, the numeric polynomial* $f^m(x)$ *totally divides by* $f^n(x)$*, that is:* $f^m(x) = f^n(x) \cdot f^{m-n}(x)$.

Consider a particular case of (1), when $n = 2$:

$$\sigma_1 + \sigma_2 = (x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2) = \sigma_2 + \sigma_1,$$
$$\sigma_1\sigma_2 = (x_1, x_2)(y_1, y_2) = [x_1(y_1 + y_2), x_2 y_1] \neq \sigma_2\sigma_1. \tag{3}$$

If the lianit analog of the complex number is chosen as $\kappa = (0, k)$, the linear equation $x + a_0 = 0$ ($a_0 \neq 0$) is not solvent. Indeed, $\sigma \cdot 1 + a_0 = (x_1, x_2)\cdot(0,1) + (0, a_0) = (x_1, 0) + (0, a_0) =$ $= (x_1, a_0) = (0,0).$

There are no solutions at $a_0 \neq 0$. The quadratic polynomial $f^2(x) = x^2 + px + q$ has a single principal lianit root ($p \neq 0$). Indeed,

$$f^2(\sigma) = \sigma^2 \cdot 1 + \sigma \cdot p + q = (x_1, x_2)[(x_1, x_2)(0,1)] + (x_1, x_2)(0, p) + (0, q) = (x_1, x_2)(x_1, 0) + (x_1 p, 0) + (0, q) =$$
$$= (x_1^2, x_1 x_2) + (x_1 p, 0) + (0, q) = (x_1^2 + x_1 p, x_1 x_2 + q) = (0,0).$$

Hence, $\sigma = (x_1, x_2) = \left(-p, \frac{q}{p}\right)$; $p \neq 0$, $q \neq 0$.

From (2) we know that the linear equaton has a solution at $\kappa = (k, 0)$, $e = (1,0)$.

The condition

$$f^2(\sigma) = \sigma^2 \cdot 1 + \sigma \cdot p + q = (0,0) \text{ leads:}$$
$$(x_1^2 + x_1 x_2, x_1 x_2) + (p x_1, p x_2) + (q, 0) =$$
$$= (x_1^2 + p x_1 + q + x_1 x_2, x_1 x_2 + p x_2) = (0,0).$$

At $x_2 \neq 0$, we have:

$\sigma = (x_1, x_2) = \left(-p, \frac{q}{p}\right)$. At $x_2 = 0$, if the equation $x_1^2 + px_1 + q = 0$ has any complex roots, the polynomial $f^2(\sigma)$ will also have secondary lianit roots $\sigma_1 = (x_{01}, 0)$, $\sigma_2 = (x_{02}, 0)$ equivalent to numeric roots, since $e = (1,0)$ is the right identity (x_{01}, x_{02} are the numeric roots to $x_1^2 + px_1 + q = 0$).

For the further we will need to construct $\sigma^m \cdot 1$, within (3), when $k = (0, k)$, m is an arbitrary integer and $\sigma = (x_1, x_2)$. We can stipulate it to be a secondary root to some polynomial $f^m(x) = x^m + a_0 x + b_0$ which yields

$$f^m(\sigma) = \sigma^m \cdot 1 + \sigma \cdot a_0 + b_0 = \sigma^m \cdot 1 + (x_1, x_2)(0, a_0) + (0, b_0) = \sigma^m \cdot 1 + (a_0 x_1, 0) + (0, b_0) = \\ = \sigma^m \cdot 1 + (a_0 x_1, b_0) = (0,0).$$

Hence,
$\sigma^m \cdot 1 = [f_1^m(x_1, x_2), f_2^m(x_1, x_2)] = (-a_0 x_1, -b_0)$,
where $f_1^m(x_1, x_2)$ and $f_2^m(x_1, x_2)$ are the elements of the lianit $\sigma^m \cdot 1$. Within (3) any $\sigma = (x_1, x_2)$ uniquely maps a quadartic polynomial $f^2(x) = x^2 + px + q = x^2 - x_1 x - x_1 x_2$, for which the lianit

$$\sigma = (x_1, x_2) = \left(-p, \frac{q}{p}\right) = \left(x_{01} + x_{02}, -\frac{x_{01} x_{02}}{x_{01} + x_{02}}\right)$$

is the principal root. According to the theorem of the principal lianit roots, the supposed numeric roots x_{01}, x_{02} of the trinomial $f^2(x) = x^2 + px + q$, coincide with the numeric roots of $f^m(x) = x^m + a_0 x + b_0$ given the condition $f^m(\sigma) = (0,0)$. Therefore,

$$\begin{cases} x_{01}^m + a_0 x_{01} + b_0 = 0 \\ x_{02}^m + a_0 x_{02} + b_0 = 0 \end{cases} \Rightarrow a_0 = \frac{x_{01}^m - x_{02}^m}{x_{02} - x_{01}}; \; b_0 = \frac{x_{01} \cdot x_{02}}{x_{01} - x_{02}} \left(x_{01}^{m-1} - x_{02}^{m-1}\right). \tag{4}$$

Hence, for the $\sigma^m \cdot 1$, we obtain

$$\sigma^m \cdot 1 = (-a_0 x_1, -b_0) = \left[f_1^m(x_1, x_2), f_2^m(x_1, x_2)\right] = \\ = \left[x_1 \cdot \frac{x_{01}^m - x_{02}^m}{x_{01} - x_{02}}, \frac{x_{01} \cdot x_{02}}{x_{01} - x_{02}} \left(x_{02}^{m-1} - x_{01}^{m-1}\right)\right]. \tag{5}$$

Using binominal coefficients and taking into account that $x_1 = x_{01} + x_{02}$, $x_2 = -\frac{x_{01} x_{02}}{x_{01} + x_{02}}$, for the first element $\sigma^m \cdot 1$ we receive $f_1^m(x_1, x_2) = \sum C_i^{m-i-1} \cdot x_1^{m-i} \cdot x_2^i$. If m is an even number, then $i = 0,1,2,\ldots,\frac{m-2}{2}$ (the number of summands $\frac{m}{2}$). If m is an odd number, then $i = 0,1,2,\ldots,\frac{m-1}{2}$ (the number of summands $\frac{m+1}{2}$). Expanded,

$$f_1^m(x_1, x_2) = x_1^m + \frac{m-2}{1!} x_1^{m-1} \cdot x_2 + \frac{(m-3)(m-4)}{2!} x_1^{m-2} \cdot x_2^2 + \\ + \frac{(m-4)(m-5)(m-6)}{3!} x_1^{m-3} \cdot x_2^3 + \frac{(m-5)(m-6)(m-7)(m-8)}{4!} x_1^{m-4} x_2^4 + \\ + \frac{(m-6)(m-7)(m-8)(m-9)(m-10)}{5!} x_1^{m-5} \cdot x_2^5 + \cdots \tag{6}$$

Using the algebra (3), for the second element of the lianit $\sigma^m \cdot 1$, we have: $f_2^m(x_1,x_2) = x_2 \cdot f_1^{m-1}(x_1,x_2)$.

Remarkably, formula (6) is a consequence of the multiplication algorithm $\sigma^m \cdot 1 = \sigma[\sigma \cdots (\sigma \cdot 1)]$ of lianits regardless the existence of numeric roots of the trinomial $f^2(x) = x^2 + px + q = x^2 - x_1 x - x_1 x_2$.

An analoguous formula can be found for $k=(k,0)$ obviously distinct from (6). Generally speaking, such formula can be found for any lianit algebra which is commutative by summation and distributive by multiplication. For the further we will need a special value of $f_2^m(x_1,x_2)$ at $x_2 = -x_1/4$. From (6) it follows

$$f_2^m\left(x_1, -\frac{x_1}{4}\right) = x_1^m \cdot \sum_i C_i^{m-i-1}\left(-\frac{1}{4}\right)^i = \begin{cases} \left(\dfrac{\frac{m}{2}}{4^{\frac{m}{2}-1}}\right) \cdot x_1^m, & \text{if } m \text{ is even} \\ \left(\dfrac{m}{4^{\frac{m-1}{2}}}\right) \cdot x_1^m, & \text{if } m \text{ is odd} \end{cases} \tag{7}$$

The First Proof to the Fundamental Theorem

Theorem (The Fundamental Theorem of Algebra). *The* n *th-degree polynomial* $f^n(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$ *with complex coefficients* a_i, $i = 1,2,\ldots,n$, $n1$, $a_n \neq 0$ *has exactly* n *complex roots* $x_{01}, x_{02}, \ldots, x_{0n}$.

Proof. Treat $f^n(x)$ within the lianit set (3), where the analog of complex number is $k=(0,k)$. Since in (3), any fixed pair of complex numbers x_1^ℓ, x_2^ℓ ($\ell = 1,2,\ldots$) uniquely defines a trinomial $f_\ell^2(x) = x^2 + p_\ell x + q_\ell = x^2 - x_1^\ell \cdot x - x_1^\ell x_2^\ell$, the principal lianit root to which is $\sigma_\ell = \left(x_1^\ell, x_2^\ell\right) = \left(-p_\ell, \dfrac{q_\ell}{p_\ell}\right)$, the condition $f^n(\sigma) = \sigma^n \cdot 1 + \sigma^{n-1} \cdot a_1 + \cdot + \sigma \cdot a_{n-1} + a_n = (0,0)$ is equivalent to searching for all possible secondary lianit roots to $f^n(\sigma)$. In terms of x_1^ℓ, x_2^ℓ the equation $f^n(\sigma) = (0,0)$ is a system of of two non-linear equations compiled according to (6). Note that the polynomial $f^n(x)$ uniquely matches the condition $f^n(\sigma) = (0,0)$.

Assume that in the general case $n > 2$, $a_n \neq 0$, the original numeric polynomial $f^n(x)$ does not have any numeric roots. This may lead to one of the two possibilities:

1. For the given set of coefficients a_i ($i = 1,2,\ldots,a_n$), the system of equations $f^n(\sigma) = (0,0)$ is trivially imcompatible (as is the case for the linear equation $x - x_0 = 0$; $x_0 \neq 0$).

2. Utilizing a sequential exclusion of x_2 ($x_1 \neq 0$, $x_2 \neq 0$, $a_n \neq 0$), we will obtain a rational-fractional expression $x_2 = \varphi_1(x_1)$, rendering the system $f^n(\sigma) = \sigma^n \cdot 1 + \sigma^{n-1} \cdot a_1 + \cdots + \sigma \cdot a_{n-1} + a_n = (0,0)$ into:

$$\begin{cases} F_1(x_1) = z_1(x_1) \cdot F_{01}(x_1) \\ F_2(x_1) = z_1(x_1) \cdot F_{02}(x_1). \end{cases} \tag{8}$$

where $F_{01}(x_1)$, $F_{02}(x_1)$ in (9) are some polynomials, $z_1(x_1)$ can be obtained using the resultant of the system $f^n(\sigma) = (0,0)$ (see for example [12]). Since we will be discussing polynomials like $z_1(x_1)$ quite often in the text we refer to them as *associated polynomial*. Indeed, the non-existence of numeric roots x_{0i} ($i = 1,2,\ldots$) for the original $f^n(x)$ excludes the possibility of any secondary lianit root $\sigma_1 = \left(x_1^1, x_2^1\right) = \left[x_1^1, \varphi_1(x_1^1)\right]$ within the algebra (3), at the chosen lianit analog of complex number $k = (0,k)$. The existence of a pair of numbers $x_1^1, x_2^1 = \varphi_1(x_1^1)$ as possible solution to the system $f^n(\sigma) = (0,0)$ is equivalent to the existence of a trinomial $f_1^2(x) = x^2 + p_1 x + q_1 = x^2 - x_1^1 \cdot x - x_1^1 \cdot x_2^1$, the numeric roots of which are those of $f^n(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$ as well. Thus the initial condition of non-existence of numeric roots at $f^n(x)$ excludes the possibility of a secondary lianit root of a possible structure $\sigma_1 = [x_1^1, \varphi_1(x_1^1)] = \left[x_{01} + x_{02}, -\frac{x_{01} \cdot x_{02}}{x_{01} + x_{02}} \right]$, where x_{01}, x_{02} are the would-be numeric roots to $f_1^2(x) = x^2 - x_1^1 \cdot x - x_1^1 \cdot \varphi_1(x_1^1)$. We conclude therefore, that if the polynomial $f^n(x)$ does not have a single numeric root then either the system $f^n(\sigma) = (0,0)$ is trivially imcompatible or the associated polynomial $z_1(x_1)$ does not have numeric roots x_1^ℓ ($\ell = 1,2,\ldots$), as well which is to say it may not have factors like $x_1 - x_1^1, x_1 - x_1^2, \ldots$, where x_1^ℓ are the would be numeric roots to $z_1(x_1)$.

Let now $f_0^m(x) = x^m + b_1 x^{m-1} + \cdots + b_{m-1}x + b_m$ ($b_m \neq 0$) be any other polynomial over the algebra (3) at a chosen $k = (0,k)$. Evidently, the system $f_0^m(\sigma) = (0,0)$ for the search of secondary lianit roots has the same properties as $f^n(\sigma) = (0,0)$. Therefore, the system for the search of possible secondary lianit roots for the polynomial $G^{m+n}(x) = f^n(x) \cdot f_0^m(x) = x^{m+n} + (a_1 + b_1)x^{m+n-1} + \cdots + a_n \cdot b_m$, that is $G^{m+n}(\sigma) = (0,0)$, must be of some degenrarate form containing $f^n(\sigma) = (0,0)$ and $f_0^m(\sigma) = (0,0)$.

For example, if one takes $f_0^m(x) = const \neq 0$ the systems for the search of secondary lianit roots for $f^n(x)$ and $const \cdot f^n(x)$ within the algebra (3) at $k = (0,k)$ have the same form.

If one assumes that $f_0^m(x)$ does have numeric roots as some $y_{01}, y_{02}, \ldots$. Then the system of equations $G^{m+n}(\sigma) = (0,0)$, for the polynomial $G^{m+n}(x) = f^n(x) \cdot f_0^m(x)$, will simpy mirror the systems $f_0^m(\sigma) = (0,0)$ and $f^n(\sigma) = (0,0)$, as the set (3) selects all possible combinations of roots $\{y_{0i}, y_{0j}\}$ ($i \neq j$), which correspond to secondary lianit roots $\sigma_\ell = (y_1^\ell, y_2^\ell) = \left(y_{0i} + y_{0j}, -\frac{y_{0i} y_{0j}}{y_{0i} + y_{0j}} \right)$ common both for $f_0^m(\sigma) = (0,0)$ and $G^{m+n}(\sigma) = (0,0)$. Therefore the generation of new types of combinations such as $\{x_{0i}, y_{0j}\}$ is possible only and only when the original polynomial $f^n(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$ has at least one numeric root distinct from the numeric roots y_{0i} of the polynomial $f_0^m(x)$.

If such roots $x_{01}, x_{02}, \ldots$ for the polynomial $f^n(x)$ are not possible, then the existence of secondary lianit roots of a structure $\sigma_\ell = \left(x_{0i} + y_{0j}, -\frac{x_{0i} y_{0j}}{x_{0i} + y_{0j}} \right)$, in the system $G^{m+n}(\sigma) = (0,0)$ is not possible. (There can be no such secondary roots in the systems $f^n(\sigma) = (0,0)$ or $f_0^m(\sigma) = (0,0)$ either).

If in the general case the original polynomial $f^n(x)$ does not have any numeric roots then there does not exist any polynomial $f_0^m(x) = x^m + b_1 x^{m-1} + \cdots + b_{m-1}x + b_m$ providing for the system $G^{m+n}(\sigma) = (0,0)$ an associated polynomial with dividors other than $z_1(x_1)$ and $u_1(x_1)$, where $z_1(x_1)$, $u_1(x_1)$ are the associated polynomials for $f^n(\sigma) = (0,0)$ and $f_0^m(\sigma) = (0,0)$.

Consider now $G^{2n}(x) = f^n(x) \cdot f^n(x)$ i.e. the case $f_0^m(x) = f^n(x)$)

$$
\begin{aligned}
G^{2n}(x) = [f^n(x)]^2 = [x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n]^2 = \\
x^{2n} + 2a_1 x^{2n-1} + (a_1^2 + 2a_2)x^{2n-2} + \cdots + 2a_{n-1} \cdot a_n x + a_n^2;
\end{aligned}
\tag{9}
$$

Since the polynomial $f^n(x)$ does not have any numeric roots then the polynomial $G^{2n}(x) = f^n(x)\cdot f^n(x)$ does not have any numeric roots either. This means that the system $G^{2n}(\sigma) = (0,0)$ within the algebra (3) is either incompatible or there exists some rational-fractional expression $x_2 = \varphi_2(x_1)$ rendering the system $G^{2n}(\sigma) = (0,0)$ into

$$\begin{cases} F_3(x_1) = z_2(x_1)\cdot F_{03}(x_1) \\ F_4(x_1) = z_2(x_1)\cdot F_{04}(x_1). \end{cases} \quad (10)$$

The absence of roots to $G^{2n}(x) = f^n(x)\cdot f^n(x)$ means that the associated polynomial $z_2(x_1)$, just like $z_1(x_1)$ from (7) cannot have numeric roots. Indeed, were such numeric roots x_1^i, $i = 1,2,\ldots$ possible within the algebra (3), the system $G^{2n}(\sigma) = (0,0)$ would have been compatible. Therefore, in its own turn the lianit polynomial $G^{2n}(\sigma) = f^n(\sigma)\cdot f^n(\sigma)$ would have had some secondary lianit roots $\sigma_i = \left(x_1^i, x_2^i\right) = \left[x_1^i, \varphi_2\left(x_1^i\right)\right]$, which is equivalent to the existence of trinomials $f_i^2(x) = x^2 + p_i x + q_i = x^2 - x_1^i x - x_1^i\cdot\varphi_2\left(x_1^i\right)$.

But then, based on the theorem of principal lianit roots, the numeric roots of such trinomials $f_i^2(x)$ would have been numeric roots to $G^{2n}(x) = f^n(x)\cdot f^n(x)$ as well, since the secondary lianit roots $\sigma_i = \left[x_1^i, \varphi_2\left(x_1^i\right)\right]$ for $G^{2n}(x)$, are at the same time principal lianit roots for $f_i^2(x) = x^2 - x_1^i\cdot x - x_1^i\cdot\varphi_2\left(x_1^i\right)$.

Whereas, the polynomial $G^{2n}(x) = f^n(x)\cdot f^n(x)$, according to our condition, does not have any numeric roots x_{0i} ($i = 1,2,\ldots$). This means, that even if the general representations (9), (10) do exist, the associated polynomial $z_2(x_1)$ either has the sole dividor of $z_1(x_1)$, or has nothing in common with the associated polynomial $z_1(x_1)$ of the system $f^n(\sigma) = (0,0)$.

We show, however, that for any $f^n(x)$, $n1$, $a_n \neq 0$, the system for the search of secondary lianit roots for $G^{2n}(x) = f^n(x)\cdot f^n(x)$, that is the system of equations $G^{2n}(\sigma) = (0,0)$ can always be presented in the form (10). Moreover the common associated polynomial $z_2(x_1)$ for any set of coefficients a_i ($i = 1,2,\ldots,n$) always has dividors.

Indeed, if we let $x_2 = -\frac{1}{4}x_1$ in the system $G^{2n}(\sigma) \equiv f^{2n}(\sigma) = (0,0)$, then using (6)-(7) the resulting single-variable system $f^{2n}(\sigma) = \sigma^{2n}\cdot 1 + \sigma^{2n-1}\cdot 2a_1 + \cdots + \sigma\cdot 2a_n\cdot a_{n-1} + a_n^2 = (0,0)$ is brought to the form

$$-3pt\begin{cases} F_5(x_1) = \left[x_1^n + 2^1\cdot a_1 x_1^{n-1} + 2^2\cdot a_2\cdot x_1^{n-2} + \cdots + 2^{n-1}\cdot a_{n-1}\cdot x_1 + 2^n\cdot a_n\right]\cdot F_{05}(x_1) \\ F_6(x_1) = \left[x_1^n + 2^1\cdot a_1 x_1^{n-1} + 2^2\cdot a_2\cdot x_1^{n-2} + \cdots + 2^{n-1}\cdot a_{n-1}\cdot x_1 + 2^n\cdot a_n\right]\cdot F_{06}(x_1). \end{cases} \quad (11)$$

If we implement the substitution $x_2 = -\frac{1}{4}x_1$ in the intial system $f^n(\sigma) = \sigma^n\cdot 1 + \sigma^{n-1}\cdot a_1 + \cdots + \sigma\cdot a_{n-1} + a_n = (0,0)$, in the general case an incompatible system will be received. The factual existence of the system (11) with an associated polynomial

$$z_0(x_1) = x_1^n + 2^1\cdot a_1 x_1^{n-1} + 2^2\cdot a_2\cdot x_1^{n-2} + \cdots + 2^{n-1}\cdot a_{n-1}\cdot x_1 + 2^n\cdot a_n,$$

proves, that for any $f^n(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$, defined within the two-element lianit set (3), the representations (7), (10), for the respective systems $f^n(\sigma) = (0,0)$, $G^{2n}(\sigma) = f^{2n}(\sigma) = (0,0)$, do exist. In a way that is quite natural if one considers that the system $f^{2n}(\sigma) = (0,0)$ for the polynomial $f^{2n}(x) = f^n(x)\cdot f^n(x)$, can never be compatible if the system $f^n(\sigma) = (0,0)$ for the $f^n(x)$ is not in the first place.

In the same way the associated polynomial $z_0(x_1) = x_1^n + 2^1\cdot a_1 x_1^{n-1} + 2^2\cdot a_2\cdot x_1^{n-2} + \cdots + 2^{n-1}\cdot a_{n-1}\cdot x_1 + 2^n\cdot a_n$

can neither be identical to the associated polynomail $z_1(x_1)$ for the system $f^n(\sigma)=(0,0)$, nor be its dividor, since it is incompatible at $x_2=-\frac{1}{4}x_1$. Along the same lines $z_0(x_1)$ cannot be identical to $z_2(x_1)$ the associated polynomial for $f^{2n}(\sigma)=(0,0)$, since as it was shown earlier, $z_1(x_1)$ must be divided by $z_2(x_1)$.

Therefore, the associated polynomial $z_2(x_1)$ which is a generating function for the secondary lianit roots for $G^{2n}(x)=f^n(x)\cdot f^n(x)$, holds in the general case at least two mutually simple dividors, that is $z_1(x_1)$ and

$$z_0(x_1)=x_1^n+2^1\cdot a_1x_1^{n-1}+2^2\cdot a_2\cdot x_1^{n-2}+\dots+$$
$$+2^{n-1}\cdot a_{n-1}\cdot x_1+2^n\cdot a_n.$$

Yet on the other hand according to the condition of the absance of numeric roots for $f^n(x)=x^n+a_1x^{n-1}+\dots+a_{n-1}x+a_n$, the polynomial $z_2(x_1)$ cannot have any other dividors but $z_1(x_1)$. The obtained contradiction proves the theorem.

Thus, there exists at least one couple of complex numbers x_1^1 and $x_2^1=-\frac{1}{4}x_1^1$, that being the solutions to the system

$$f^{2n}(x)=f^n(x)\cdot f^n(x),$$

i.e. $f^{2n}(\sigma)=(0,0)$, render it into the compatible form of (11) with the associated polynomial $z_0(x_1)=x_1^n+2^1\cdot a_1x_1^{n-1}+2^2\cdot a_2\cdot x_1^{n-2}+\dots+$
$+2^{n-1}\cdot a_{n-1}\cdot x_1+2^n\cdot a_n$.

The associated polynomial for the system $f^n(\sigma)=(0,0)$ with respect to x_2 reads as

$$z_0(x_2)=x_2^n-\frac{1}{2^1}\cdot a_1x_2^{n-1}+\frac{1}{2^2}\cdot a_2\cdot x_2^{n-2}-\dots+$$
$$+(-1)^n\cdot\frac{1}{2^n}\cdot a_n.$$

Therefore, within (3), at $k=(0,k)$, the lianit $\sigma_1=\left(x_1^1,x_2^1\right)=\left(x_1^1,-\frac{1}{4}x_1^1\right)$ is a secondary root to $f^{2n}(x)=f^n(x)\cdot f^n(x)$, being at the same time the principal lianit root for some trinomial

$$f_1^2(x)=x^2+p_1x+q_1=x^2-x_1^1x-x_1^1\cdot x_2^1=x^2-x_1^1x-x_1^1\cdot\left(-\frac{1}{4}x_1^1\right)=x^2-x_1x+\frac{x_1^2}{4}=\left(x-\frac{x_1^1}{2}\right)^2.$$

Based on the theorem of principal lianit roots the numeric roots of $x_{01}=\frac{x_1^1}{2}$ and $x_{02}=x_{01}=\frac{x_1^1}{2}$ of that trinomial $f_1^2(x)=x^2-x_1x+\frac{x_1^2}{4}$ are at the same time the roots of the polynomial $f^{2n}(x)=f^n(x)\cdot f^n(x)$ (x_1^1 is the numeric root of $z_0(x_1)$). Evidently the existence of at least one numeric root $x_{01}=\frac{x_1^1}{2}$ for the original polynomial $f^n(x)=x^n+a_1x^{n-1}+\dots+a_{n-1}x+a_n$, gurantees the exisence of n numeric roots $x_{01},x_{02},\dots,x_{0n}$.

Because the algebra (3), with $k=(0,k)$, can involve only two-root combinations $\{x_{0i},x_{0j}\}$ ($i\neq j$), the structure of each secondary lianit root of the original polynomial $f^n(x)$ is determined thus: $\sigma_\ell=\left(x_1^\ell,x_2^\ell\right)=\left(x_{0i}+x_{0j},-\frac{x_{0i}x_{0j}}{x_{0i}+x_{0j}}\right)$.

Indeed, every possible combination $\{x_{0i},x_{0j}\}$ defines uniquely a quadratic trinominal $f_\ell^2(x)=x^2+p_\ell x+q_\ell=x^2-x_1^\ell\cdot x-x_1^\ell x_2^\ell$, for which the lianit $\sigma_\ell=\left(x_1^\ell,x_2^\ell\right)=\left(-p_\ell,\frac{q_\ell}{p_\ell}\right)$ is a principal root. Therefore, the degree of the general associated polynomial $z_1(x_1)$ of the system $f^n(\sigma)=(0,0)$ can be calculated as $n_0=C_2^n\cdot 1=\frac{n(n-1)}{2}$.

The numeric roots of that polynomial are all possible $x_1^\ell = x_{0i} + x_{0j}$; ($i \neq j$, $\ell = 1,2,\ldots,\frac{n(n-1)}{2}$).

The roots of $G^{2n}(x) = f^n(x)\cdot f^n(x)$ are given as $x_{01}, x_{02},\ldots,x_{0n}; x_{01}, x_{02},\ldots,x_{0n}$. Consequently, the number of possible secondary roots equals to the number of all possible combinations $C_2^{2n}\cdot 1 = n(2n-1)$. Among them, the number of combinations of the form $\{x_{0i}, x_{0j}\}$ ($i \neq j$), is $n(2n-1) - n = \frac{n(n-1)}{2}\cdot 4$ in total.

This means, that the associated polynomial $z_2(x_1)$ of the system deriving the secondary lianit roots $f^{2n}(\sigma) = (0,0)$ must have a dividor $\left[z_1^{\frac{n(n-1)}{2}}(x_1)\right]^4$.

At the same time there are exactly n combinations such $\{x_{0i}, x_{0i}\}$ ($i = 1,2,\ldots,n$), each one definining a secondary lianit root

$$\sigma_i = \left(x_1^i, x_2^i\right) = \left(x_{0i} + x_{0i}, -\frac{x_{0i}\cdot x_{0i}}{x_{0i} + x_{0i}}\right) = \\ = \left(2x_{0i}, -\frac{x_{0i}}{2}\right) = \left(x_1^i, -\frac{1}{4}x_1^i\right).$$

The appearance of a second dividor for the associated polynomial $z_2(x_1)$ in the system of secondary lianit roots $f^{2n}(\sigma) = (0,0)$ is a result of these additional n lianit roots. That is, $z_0^n(x_1) = x_1^n + 2^1\cdot a_1 x_1^{n-1} + 2^2\cdot a_2\cdot x_1^{n-2} + \cdots + 2^{n-1}\cdot a_{n-1}\cdot x_1 + 2^n\cdot a_n$ the numeric roots of which are $x_1^i = 2x_{01}, 2x_{02},\ldots,2x_{0n}$.

Obviously, if we consider $f^{n\ell_0}(x) = \left[f^n(x)\right]^{\ell_0}$, where $\ell_0 = 3,4,5,\ldots$, will have same combinations $\{x_{0i}, x_{0j}\}$ ($i \neq j$) and $\{x_{0i}, x_{0i}\}$. Consequently, the general associated polynomial $z(x_1)$ for the system $f^{n\ell_0}(\sigma) = (0,0)$ can be written as: $z(x_1) = \left[z_0^n(x_1)\right]^{\frac{\ell_0(\ell_0-1)}{2}}\cdot\left[z_1(x_1)\right]^{\ell_0^2}$.

The same result can be obtained if one inserts into the system $f^{n\ell_0}(\sigma) = (0,0)$ the substitution $x_2 = -x_1/4$.

It may be that the degree of the associated polynomial $z_1(x_1)$ is lower than $n_0 = n(n-1)/2$. It manifests in the existence of numeric roots $\pm x_{0i}$ in the original $f^n(x)$, however the degree and the structure of the associated polynomial $z_0^n(x_1) = x_1^n + 2^1\cdot a_1 x_1^{n-1} + \cdots + 2^{n-1}\cdot a_{n-1}\cdot x_1 + 2^n\cdot a_n$ do not depend on that.

Obviously, if one replaces $k = (0,k)$ by $k = (k,0)$, in the systems $f^n(\sigma) = (0,0)$ and $f^{2n}(\sigma) = (0,0)$ respectively, will appear along with the associated polynomials $z_1^{n(n-1)/2}(x_1)$ and $z_0^n(x_1) = x_1^n + 2^1\cdot a_1 x_1^{n-1} + \cdots + 2^{n-1}\cdot a_{n-1}\cdot x_1 + 2^n\cdot a_n$, an associated polynomial corresponding to combinations $\{x_{0i}\}$, that is: $u_0^n(x_1) = x_1^n + a_1 x_1^{n-1} + \cdots + a_{n-1}\cdot x_1 + \cdot a_n$ (at $k = (k,0)$, the equation $x - x_{0i} = 0$ is solvable). The Theorem is proved.

The Second Proof to the Fundamental Theorem

In this section we prove the FTA using another algebra

$$\begin{aligned} &\sigma_1 + \sigma_2 = (x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2); \\ &\sigma_1\cdot\sigma_2 = (x_1, x_2)\cdot(y_1, y_2) = (x_1 y_1 \pm x_2 y_2, x_1 y_2 + x_2 y_1);\ k = (k,0). \end{aligned} \tag{12}$$

Algebra (12) is commutative, associate and distributive with respect to addition and multiplication, $k = (k,0)$ figures as a complex number (at the sign $(-)$, we obtain the algebra of complex numbers). Some properties of (12) with respect to lianit and numeric roots are discussed in the Appendix.

We are required to prove that for the given polynomial $f^n(x)$ there exists at least one polynomial $f_0^m(x)$ such that the system of equations required for the secondary lianit solutions for the adjoined polynomial $G^{m+n}(x) = f^n(x)\cdot f_0^m(x)$ is compatible. At that,

the common associated polynomial for that system must have dividors with respect to either x_1 or x_2. As a "probing" polynomial $f_0^m(x)$ we take $f_0^n(x) = x^n + 5\cdot a_1 \cdot x^{n-1} + 5^2 \cdot a_2 x^{n-2} \cdots +$
$+5^{n-1}\cdot a_{n-1}x + 5^n \cdot a_n$.

We realize that if $f^n(x)$ indeed has numeric roots $x_{01}, x_{02}, \ldots, x_{0n}$, then the roots of $f_0^n(x)$ are also existent as $5x_{01}, 5x_{02}, \ldots, 5x_{0n}$. On the other hand, our knowledge of the structure of secondary lianit roots of the system $G^{2n}(\sigma) = (0,0)$ for the polynomial $G^{2n}(x) = f^n(x)\cdot f_0^n(x)$ indicates that along with possible sets like $\{x_{0i}\}$; $\{5x_{0i}\}$, $\{x_{0i}, x_{0j}\}$; $\{5x_{0i}, 5x_{0j}\}$; $\{x_{0i}, 5x_{0j}\}$; $(i \neq j)$ there must be permitted also such sets as $\{x_{0i}, 5x_{0i}\}$.

This hints that $G^{2n}(\sigma) = f^n(\sigma)\cdot f_0^n(\sigma)$ must have some additional secondary roots which are different from those obtained from $f^n(\sigma) = (0,0)$, $f_0^n(\sigma) = (0,0)$. Since the possible structure of such roots must be

$$\sigma_i = \left(x_1^i, x_2^i\right) = \left[\frac{x_{0i}+5x_{0i}}{2}, \pm\left(\frac{x_{0i}-5x_{0i}}{2}\right)\right] = \\ = \left[3x_{0i}, \pm 2x_{0i}\right];$$

$i = 1,2,\ldots,n$ it motivates us to make the substitution $x_2 = \pm\frac{2}{3}x_1$ and expect that the system $G^{2n}(\sigma) = (0,0)$ will be compatible with respect to x_1 with an associated polynomial $z_0^n(x_1) = x_1^n + 3^1\cdot a_1 \cdot x_1^{n-1} + 3^2 \cdot a_2 \cdot x_1^{n-2} + \cdots +$
$+3^{n-1}\cdot a_{n-1}\cdot x_1 + 3^n \cdot a_n$.

The elements of $f^n(\sigma)$ are calculated in the Appendix (see the formula (15)). Using the relations in (15) one sees that the said substitution makes the system $G^{2n}(\sigma) = (0,0)$ compatible with an associated polynomial $z_0^n(x_1)$. Of course, a variety of other probing polynomials can be used. Take for example

$$f_0^n(x) = x^n + x_0 a_1 x^{n-1} + x_0^2 a_2 x^{n-2} + \cdots + x_0^n a_n,$$

where x_0 is any non-trivial complex number.

The existence of $z_0^n(x_1)$ shows that the system $G^{2n}(\sigma) = (0,0)$ is compatible with a general associated polynomial $z(x_1)$ of degree $N_1 = C_1^{2n}\cdot 1 + C_2^{2n}\cdot 1$. With respect to x_2, from (14) we have $N_2 = C_1^{2n}\cdot 1 + C_2^{2n}\cdot 2$. Therefore, $z_0^n(x_1) = x_1^n + 3^1\cdot a_1\cdot x_1^{n-1} + 3^2\cdot a_2 \cdot x_1^{n-2} + \cdots +$
$+3^{n-1}\cdot a_{n-1}\cdot x_1 + 3^n \cdot a_n$
cannot coincide with $z(x_1)$ but is a dividor of it.

The proof of the main theorem in the calculative aspect can also be well shown using a probing function $G^{n+1}(x) = x\cdot f^n(x)$. If the original numeric polynomial $f^n(x)$ did not have numeric roots the pseudo-polynomial $G^{n+1}(\sigma)$ is not permitted to have secondary roots of the structure

$$\sigma_i = \left(x_1^i, x_2^i\right) = \left(\frac{x_{0i}+0}{2}, \pm\frac{x_{0i}-0}{2}\right) = \left(x_1, \pm x_1\right).$$

Therefore, the system $G^{n+1}(\sigma) = 0$ after the substitution $x_2 = \pm x_1$ could not be compatible. Using the results obtained above one can easily verify that such a substitution leads to the existence of an associated polynomial

$$z_0(x_1) = x_1^n + \frac{a_1}{2}x_1^{n-1} + \frac{a_2}{2^2}x_1^{n-2} + \cdots + \frac{a_{n-1}}{2^{n-1}}x_1 + \frac{a_n}{2^n}, \tag{13}$$

which proves the incorrectness of our initial assumption of the absence of roots for the n-th degree numeric algebraic polynomial $f^n(x)$.

Summary

To conclude we summarize the results of our work. We have provided a purely algebraic method of proving the Fundamental Theorem of Algebra based on the newly introduced formalism of lianit algebras.

Utilizing the theorems of principal and secondary lianit roots of pseudo-polynomials we were able to establish a unique connection between the existence of numeric roots of numeric polynomials and the existence of secondary roots of a special auxiliary pseudo-polynomial which for the sake of conveniency we have called a probing polynomial. Thus, the existence of numeric roots is rendered into the existence of secondary lianit roots which within the developed formalism is a simple calculative algebraic task. By constructing two concrete examples of lianit algebras and assuming the absence of complex roots for a numeric polynomial of arbitrary degree we were able to arrive at a contradiction.

Appendix

In Appendix we study some of the basic calculative properties of lianit algebras introduced in the main text. A dedicated text covering all the applications can be found in the Dissertation [11].

Consider first the algebra (12). The linear equation $x - x_{0i} = 0$ has a single principal lianit root $\sigma = (x_{0i}, 0)$.

The trinomial $f^2(x) = x^2 + px + q$ has two principal lianit roots. Indeed,

$$f^2(\sigma) = \left(x_1^2 + x_2^2, 2x_1x_2\right) + (px_1, px_2) + (q,0) = \\ = \left(x_1^2 + x_2^2 + px_1 + q, 2x_1x_2 + px_2\right) = (0,0).$$

Hence:

$$\sigma_1 = \left(x_1^1, x_2^1\right) = \left(-\frac{p}{2}, +\sqrt{\frac{p^2}{4} - q}\right); \quad \sigma_2 = \left(x_1^2, x_2^2\right) = \left(-\frac{p}{2}, -\sqrt{\frac{p^2}{4} - q}\right). \tag{14}$$

The specific property of (12) is that for any given $f^n(x) = x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$, the sum $x_{0\ell} = x_1^\ell + x_2^\ell$ (l numerates the lianit root) of elements of its secondary lianit roots $\sigma_\ell = \left(x_1^\ell, x_2^\ell\right)$ is a numeric root to $f^n(x)$. Indeed, for $\sigma^n = \left[f_1^n(x_1, x_2); f_2^n(x_1, x_2)\right]$ we have:

$$-15pt\,If\;\; n\; is\; an\; even\; number, then:$$
$$f_1^n(x_1,x_2) = C_n^n x_1^n + C_{n-2}^n x_1^{n-2}x_2^2 + \cdots + C_0^n x_2^n,$$
$$f_2^n(x_1,x_2) = C_{n-1}^n x_1^{n-1}x_2 + C_{n-3}^n x_1^{n-3}x_2^3 + \cdots + C_1^n x_1 x_2^{n-1}.$$
$$-15pt\,If\;\; n\; is\; an\; odd\; number, then:$$
$$f_1^n(x_1,x_2) = C_n^n x_1^n + C_{n-2}^n x_1^{n-2}x_2^2 + \cdots + C_1^n x_1 x_2^{n-1},$$
$$f_2^n(x_1,x_2) = C_{n-1}^n x_1^{n-1}x_2 + C_{n-3}^n x_1^{n-3}x_2^3 + \cdots + C_0^n x_2^n \tag{15}$$

where (15), C_i^n are the binomial coefficients: $C_i^n = \dfrac{n!}{i!(n-i)!}$. Compiling now $f^n(\sigma) = \sigma^n + a_1 \cdot \sigma^{n-1} + \cdots + a_{n-1} \cdot \sigma + a_n = (0,0)$ and summing up the both equations, we obtain: $(x_1 + x_2)^n + a_1(x_1 + x_2)^{n-1} + \cdots + a_{n-1}(x_1 + x_2) + a_n = 0$, which means that $x_0 = x_1 + x_2$ is a numeric root to $f^n(x)$, under the condition that within the lianit set (12) secondary roots $\sigma_\ell = \left(x_1^\ell, x_2^\ell\right)$ are existent. Following (14), these supposed roots must have the form: $\sigma_\ell = \left(x_1^\ell, x_2^\ell\right) = \left[\dfrac{x_{0i} + x_{0j}}{2}, \pm\left(\dfrac{x_{0i} - x_{0j}}{2}\right)\right]$.

Consider next the two-element lianit set

$$\sigma_1 + \sigma_2 = (x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2) = \sigma_2 + \sigma_1; \\ \sigma_1 \cdot \sigma_2 = (x_1, x_2)(y_1, y_2) = \left[x_1(y_1 + y_2) + x_2y_1, x_2y_2 - x_1y_1\right]; \; k = (0, k). \tag{16}$$

The lianit algebra (16) is commutative with respect to addition and it is associative and distributive with respect to multiplication. The element $e = (0,1)$ is the right identity. The linear and quadratic equations are solvable within (16). Therefore, based on the main theorem of principal roots, any polynomial $f^3(x) = x^3 + bx + c$ is obliged to have secondary lianit roots within (16). The condition $f^3(\sigma) = \sigma^3 + \sigma \cdot b + c = (0,0)$, assuming $x_1 \neq 0$, yields

$$\begin{cases} 3x_2^2 + 3x_1x_2 + b = 0 \\ x_2^3 + bx_2 - c - \left(x_1^3 + 3x_1^2x_2\right) = 0 \end{cases} \Rightarrow x_1 = -\left(\frac{b + 3x_2^2}{3x_2}\right) \tag{17}$$

which leads to $-27x_2^6 + 27cx_2^3 + b^3 = 0$ and hence

$$x_2 = \sqrt[3]{\frac{c}{2} \pm \sqrt{\frac{c^2}{4} + \frac{b^3}{27}}}\,. \qquad (18)$$

Consequently, there exist six secondary lianit roots for $f^3(x) = x^3 + bx + c$, which write in the form $\sigma_\ell = \left(x_1^\ell, x_2^\ell\right) = \left[-\left(\frac{b+3x_2^2}{3x_2}\right), x_2\right]$. Each of these roots corresponds uniquely to a quadratic equation within (16), the numeric roots of which coincide with the roots of $f^3(x) = x^3 + bx + c$. To find the unknown quadratic trinomials $f^2(x) = x^2 + px + q$ we write the condition $f^2(\sigma) = \sigma^2 + \sigma \cdot p + q = (0,0)$ and obtain

$$\begin{cases} x_1^2 + 2x_1x_2 + px_1 = 0 \quad (x_1 \neq 0) \\ x_2^2 - x_1^2 + px_2 + q = 0 \end{cases}. \qquad (19)$$

There is no need to determine the coefficients p and q via (18), since as we know the required values $p = -(x_1 + 2x_2)$ coincide with the roots of $f^3(x) = x^3 + bx + c$. This leads to the Cardano formula with one of the cubic radicals (18)).

From (18) the following six values for x_2 are obtained

$$x_{21} = \sqrt[3]{\frac{c}{2} + \sqrt{\frac{c^2}{4} + \frac{b^3}{27}}}\,,\ Bx_{21},\ Dx_{21}$$

$$x_{22} = \sqrt[3]{\frac{c}{2} - \sqrt{\frac{c^2}{4} + \frac{b^3}{27}}}\,,\ Bx_{22},\ Dx_{22},$$

where B, D are the primitive roots of $x^3 - 1 = 0$. Thus, the secondary roots of $f^3(x) = x^3 + bx + c$ have the structure

$$\begin{cases} \sigma_1 = \left(-\frac{b+3x_{21}^2}{3x_{21}}, x_{21}\right);\ \sigma_2 = \left(-\frac{b+3Dx_{21}^2}{3Bx_{21}}, Bx_{21}\right);\ \sigma_3 = \left(-\frac{b+3Bx_{21}^2}{3Dx_{21}}, Dx_{21}\right); \\ \sigma_4 = \left(-\frac{b+3x_{22}^2}{3x_{22}}, x_{22}\right);\ \sigma_5 = \left(-\frac{b+3Dx_{22}^2}{3Bx_{22}}, Bx_{22}\right);\ \sigma_6 = \left(-\frac{b+3Bx_{22}^2}{3Dx_{22}}, Dx_{22}\right). \end{cases} \qquad (20)$$

We now demonstrate how the explicit expressions for the cubic polynomial $f^3(x) = x^3 + ax^2 + bx + c$ can be obtained in the general case using now the principal lianit roots. To this end we construe the following lianit algebra

$$\begin{aligned} &\sigma_1 \cdot \sigma_2 = (x_1, x_2, x_3)(y_1, y_2, y_3) = \\ &= [x_1y_1 + x_2y_3 + x_3y_2, x_3y_3 + x_1y_2 + x_2y_1, x_2y_2 + x_1y_3 + x_3y_1); \\ &k = (k,0,0). \end{aligned} \qquad (21)$$

The algebra(21) is commutative, associative and distributive with a unity element $e = (1,0,0)$. The condition

$f^3(\sigma) = \sigma^3 + a\sigma^2 + b\sigma + c = (0,0,0)$,

gives

$$\begin{cases} x_1^3 + x_2^3 + x_3^3 + a\left(x_1^2 + 2x_2x_3\right) + 6x_1x_2x_3 + bx_1 + c = 0, \\ 3x_2^2x_3 + 3x_3^2x_1 + 3x_1^2x_2 + a\left(x_3^2 + 2x_1x_2\right) + bx_2 = 0, \\ 3x_3^2x_2 + 3x_2^2x_1 + 3x_1^2x_3 + a\left(x_2^2 + 2x_1x_3\right) + bx_3 = 0. \end{cases} \qquad (22)$$

The principal roots are obtained from the condition

$$x_1 = -\frac{a}{3},\ x_2 \neq x_3,\ x_2x_3 = \frac{a^2 - 3b}{9},\ x_2^3 + x_3^3 = \frac{9ab - 2a^3 - 27c}{27}. \qquad (23)$$

The case $x_2 = x_3$ leads to secondary roots. Adding the equations of the system (22) we reach $(x_1 + x_2 + x_3)^3 + a(x_1 + x_2 + x_3)^2 + b(x_1 + x_2 + x_3) + c = 0$. In other words, the number $x_0 = x_1 + x_2 + x_3$ is a numeric root. Calculating the x_2, x_3 via the (23) one obtains a general expression for the numeric roots of $f^3(x) = x^3 + ax^2 + bx + c$. In particular, at $a = 0$ ($x_1 = -\frac{a}{3} = 0$), we have $x_3 = -\frac{b}{3x_2}$,

$x_2^6 + cx_2^3 - \frac{b^3}{27} = 0$, Introducting the denotions x_{21}, x_{21}

$$x_{21} = \sqrt[3]{-\frac{c}{2} + \sqrt{\frac{c^2}{4} + \frac{b^3}{27}}}\,, \qquad x_{22} = \sqrt[3]{-\frac{c}{2} - \sqrt{\frac{c^2}{4} + \frac{b^3}{27}}}\,, \quad \text{we obtain } x_0 = x_{21} + x_{22}. \tag{24}$$

The explicit expressions for all the six principal lianit roots read:

$$\begin{cases} \sigma_1 = \left(0, x_{21}, -\frac{b}{3x_{21}}\right); \ \sigma_2 = \left(0, Bx_{21}, -\frac{b}{3Bx_{21}},\right); \ \sigma_3 = \left(0, Dx_{21}, -\frac{b}{3Dx_{21}},\right); \\ \sigma_4 = \left(0, x_{22}, -\frac{b}{3x_{22}},\right); \ \sigma_5 = \left(0, Bx_{22}, -\frac{b}{3Bx_{22}},\right); \ \sigma_6 = \left(0, Dx_{22}, -\frac{b}{3Dx_{22}},\right). \end{cases} \tag{25}$$

where B and D are the primitive roots of $x^3 - 1$, i.e. $B, D = \frac{-1 \pm i\sqrt{3}}{2}$.

The explicit expressions for the roots to the quartics $f^4(x) = x^4 + b_0x^2 + c_0x + d_0$ are particularly easily obtained within the algebra (3), at $k = (0, k)$ i.e $\sigma_1\sigma_2 = (x_1, x_2)(y_1, y_2) = [x_1(y_1 + y_2), x_2y_1]$. The system $f^4(\sigma) = \sigma^4 \cdot 1 + \sigma^2 \cdot b_0 + \sigma \cdot c_0 + d_0 = (0,0)$ after the assumption $x_1 \neq 0$ gives

$$\begin{cases} x_1^3 + 2x_1^2x_2 + b_0x_1 + c_0 = 0 \\ x_1^3x_2 + x_1^2x_2^2 + b_0x_1x_2 + d_0 \end{cases}; x_2 = -\left(\frac{x_1^3 + b_0x_1 + c_0}{2x_1^2}\right); \tag{26}$$

which leads to Lagrange cubit resolvent [12] for the quartic equation

$$z^3 + 2b_0z^2 + \left(b_0^2 - 4d_0\right)z - c_0^2 = 0; \ (z = x_1^2). \tag{27}$$

where we have denoted $x_1^2 = z$. Thus in the set (3), the quartics $f^4(x) = x^4 + b_0x^2 + c_0x + d_0$ has six secondary lianit roots $\sigma = (x_1, x_2) = \left(x_1, -\frac{x_1^3 + b_0x_1 + c_0}{2x_1^2}\right)$. Each of these roots corresponds uniquely to a quadratic polynomial

$$f^2(x) = x^2 + px + q = x^2 - x_1 \cdot x - x_1 \cdot x_2 = = x^2 - x_1x + \left(\frac{x_1^3 + b_0x_1 + c_0}{2x_1}\right),$$

the numeric roots of which coincide with those of $f^4(x) = x^4 + b_0x^2 + c_0x + d_0$.

To conclude the Appendix we stop at the generalization of Vieta's relations in the domain of lianit algebras. This matter was studied in the work [3] and the Dissertation [11]. Here we will bring out the main result and provide illustrative examples.

In [3], [11] it was shown that if an n-element lianit algebra is commutative with respect to addition and is distributive with respect to multiplication and has a right-handed lianit analog of compex number k then some relations reminding the Vieta's relations for numeric roots are possible for lianit roots. To be precise, consider $f^n(\sigma)$ with N principal lianit roots (N may be greater or less than n or may coincide with n as is the case of secondary lianit roots). The extended Vieta's relations are manifested through all possible sums of products $\sum\sigma_i$, $\sum\sigma_i\sigma_j$, $\sum\sigma_i\sigma_j\sigma_\ell$, … being lianits the elements of which are rational functions of the coefficients a_i of the original numeric polynomial $f^n(x)$. As a vivid illustration of this, consider the algebra of three-element lianits

$$\begin{aligned} &\sigma_1 \cdot \sigma_2 = (x_1, x_2, x_3)(y_1, y_2, y_3) = \\ &= [x_1y_2 + x_2y_1, x_2y_2 - x_3y_3, x_2y_3 + x_3y_2 + x_1y_1); \\ &k = (0, k, 0) \end{aligned} \tag{28}$$

The set (28) is not associative with respect to multiplication and $e = (0,1,0)$ is its right-handed unity. The cubic $f^3(x) = x^3 + ax^2 + bx + c$ has exactly four principal lianit roots

$$x_1^2 = \pm\left(\frac{2a^3-9ab+27c}{27\sqrt{b-\frac{a^2}{3}}}\right) \neq 0; \quad x_3^2 = b-\frac{a^2}{3} \neq 0; \quad x_2 = -\frac{a}{3}. \tag{29}$$

At $a = 0$ we have

$$\begin{aligned}\sigma_1 &= \left(\sqrt{\frac{c}{\sqrt{b}}},0,\sqrt{b}\right); &\quad \sigma_2 &= \left(-\sqrt{\frac{c}{\sqrt{b}}},0,\sqrt{b}\right);\\ \sigma_3 &= \left(\sqrt{-\frac{c}{\sqrt{b}}},0,-\sqrt{b}\right); &\quad \sigma_4 &= \left(-\sqrt{-\frac{c}{\sqrt{b}}},0,-\sqrt{b}\right).\end{aligned} \tag{30}$$

Since the lianits (30) are commutative but not associative with respect to multiplication from all possible 24 products $\sigma_i\sigma_j\sigma_\ell$ one should choose the half as $\sigma_i(\sigma_j\sigma_\ell) = \sigma_i(\sigma_\ell\sigma_j)$. The same applies when construing the products $\sigma_i\sigma_j\sigma_\ell\sigma_k$ ($i, j, \ell, k = 1,2,3,4$). In detail we have

$$\begin{aligned}&\sigma_1+\sigma_2+\sigma_3+\sigma_4 = (0,0,0),\\ &\sigma_1\sigma_2+\sigma_1\sigma_3+\sigma_1\sigma_4+\sigma_2\sigma_3+\sigma_2\sigma_4+\sigma_3\sigma_4 = (0,2b,0),\\ &\sigma_1(\sigma_2\sigma_3+\sigma_2\sigma_4+\sigma_3\sigma_4)+\sigma_2(\sigma_3\sigma_4+\sigma_1\sigma_3+\sigma_1\sigma_4)+\\ &\quad+\sigma_3(\sigma_1\sigma_4+\sigma_2\sigma_4+\sigma_1\sigma_2)+\sigma_4(\sigma_1\sigma_2+\sigma_1\sigma_3+\sigma_2\sigma_3) = (0,-4c,0),\\ &\sigma_1(\sigma_2\sigma_3\sigma_4+\sigma_3\sigma_2\sigma_4+\sigma_4\sigma_2\sigma_3)+\sigma_2(\sigma_1\sigma_3\sigma_4+\sigma_3\sigma_1\sigma_4+\sigma_4\sigma_1\sigma_3)+\\ &\quad+\sigma_3(\sigma_1\sigma_2\sigma_4+\sigma_2\sigma_1\sigma_4+\sigma_4\sigma_1\sigma_2)+\sigma_4(\sigma_1\sigma_2\sigma_3+\sigma_3\sigma_1\sigma_2+\sigma_2\sigma_1\sigma_3) = (0,12b^2,0).\end{aligned} \tag{31}$$

If one now returns to the the subsets σ_1, σ_2, σ_3 and σ_4, σ_5, σ_6 in (20), one can see the fulfillment of classic Vieta relationships $\sigma_1+\sigma_2+\sigma_3 = (0,0)$;
$\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_1\sigma_3 = (0,b)$, $\sigma_1\sigma_2\sigma_3 = (0,-c)$, and the set of all six secondary roots satisfy the Vieta relationships for the polynomial $f^6(x) = (x^3+bx+c)^2$ where x is treated as a lianit. In the same way, the subset of principal roots σ_1, σ_2, σ_3 and σ_4, σ_5, σ_6 in (25), just as in the case of secondary roots (20), satisfy the system of Vieta relations for the polynomial $f^3(x) = x^3+ax^2+bx+c$

($a = 0$; $x_{22} = -b/3x_{21}$).

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RURAL ENERGY SUPPLY SYSTEMS WITH BIPV AND VACUUM INSULATING GLASS UNITS

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The work demonstrates the possibility of volume-spatial and typological decisions development for power-independent buildings taking resort-and-health-improving and educational-and-recreational complexes (RHIERC) as an example. The project is dedicated to the development of RHIERC architecture and their construction with the use of solar systems and vacuum insulating glass units. The local level and micro-level evaluation of building site energy potential for the subsequent architecturally-constructive buildings design, first of all, for mounting solar panels on the facades and roofs of buildings, have been made. The evaluation of buildings' energy potential followed by the architecturally-constructive design is incorporated into an integrated method.

Keywords: *Energy Supply of Rural Buildings, Complex Systems, Building Integrated PV (BIPV), Stand-alone Solar Systems, Vacuum Insulating Glass Unit, Solar Architecture*.

Introduction

For rural areas it is preferable to develop a town-forming sector in form of small power-independent blocks, ecovillage that could be based on RHIERC [1, 2].

Today, the construction and development in resort-and-health-improving, recreation-and-tourist industry in Russia lags behind demand, while the quality of provided services doesn't fit the required level. Resort-and-health-improving, recreation-and-tourism industry in Russia have to see significant losses due to the problems of power supply and poor infrastructure of areas adjoining reserves, natural areas under strict protection and other places of interest for eco-tourism, recreation and health care. This makes the development of the architecture of the resort-and-health-improving and educational-and-recreational objects and their construction with use of solar systems with respect to the energy-efficient solutions of building made of natural materials and proper environmental management essentially important.

At the stage of setting the objectives of design experiment the RHIERC have been chosen. In some cases, the power supply appears the major problem because the RHIERC in question are located too far from urban centers. Therefore the development of RHIERC architecture and their construction in the form of complexes comprising energy-independent buildings with integrated solar systems is the most perspective concept.

Besides, there is a problem of solar cells technological concepts application in building scales. The existing solutions are not unified and can not be adapted for mass construction applications. Neither is the esthetic point of photoelectric power technologies considered in building applications. Economic efficiency can be reached at work with RHIERC. After that, experience of application and operation will be used for large-lot industrial production for objects on urbanized territories.

Problem solution methods and results

The work demonstrates the energy supply systems creation in rural area of Russia and possibility of volume-spatial and typological decisions development for power-independent buildings taking RHIERC in ecological village GENOM as an example (set. Igodovo, Lat 58,01 Lon 43,34) [2].

GENOM means Global Ecovillage Network Office Management. In the building of the recreational and educational complex it is planned to place the headquarters of the Global Ecovillage Network (GEN) association. The GEN association has the status of a special adviser at the Economic and Social Council of the United Nations (UN-ECOSOC) and a partner of the United Nations Institute of Training and Researches (UNITAR). The GEN association principles give new opportunities in the GENOM ecovillage project as a pilot settlement on the basis of which natural territories will efficiently develop.

The project addresses the environmental problem of energy supply to facilities located far from megapolises. This is achieved by using only renewable energy sources, such as the sun and wind. The innovative potential of this work involves the use of the results for developing small and medium-size businesses in the resort and sanatorium industry and redirecting agricultural enterprises. The aim of this work in regional planning is to increase the economical potential of enterprises based on a sustainable management of natural resources. The main task for the building designers (architectural engineers) was to improve the architectural and artistic qualities of energetically autonomous facilities.

The analysis of invariably economically expedient situation was made on the basis of energy potential of buildings of the resort-and-health-improving and educational-and-recreational complexes. The evaluation of the energy potential of buildings area determines a subsequent architecture-constructional design on local and micro levels. First of all, it concerns mounting solar panels on the facades and roofs of buildings. With buildings properly oriented in the environment the energy efficiency can be provided. Using this planning concept makes it possible to save resources.

The results of RHIERC design experiment in ecovillage GENOM have helped to reveal certain steady trends in building designing aimed at a higher energy reception. First of all, a developed facade relief or a building silhouette shall be provided from the sunny side making it possible to mount a large number of solar panels capable to receive 40% to 50 % more solar energy in absolute values. The efficiency evaluation is necessary to define the most favorable tendency in various building proportions use. In the project, the strictly west-east orientation of one extended facade or the north-south orientation of two extended facades has been found most effective (Fig. 1).

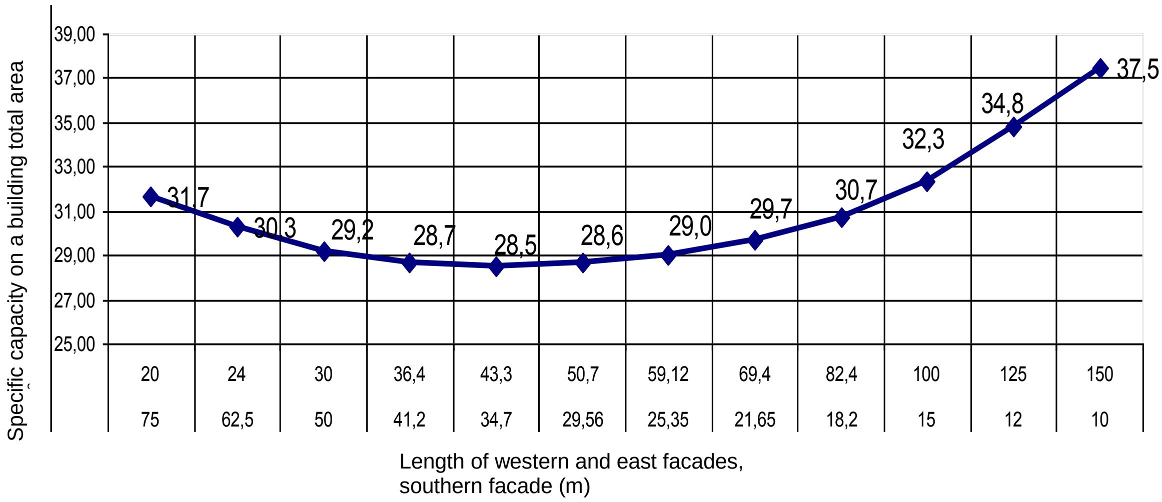


Fig. 1. Evaluation of energy efficiency of proportions of building with solar systems

While designing innovative buildings with integrated solar systems the physical parameters of the environment shall be considered with respect to providing the optimum performance and comfortable environment in premises as well as to realization of their energy potential. The physical-technical factors are analyzed to take the advantage of the energy reception feature and energy use in buildings. Thereby the power independence or, at least, energy saving of building operation is achieved.

The building orientation has been analyzed with the respect to overall equipment performance, taking the architecture meeting the functional requirements of the whole complex into account (Fig. 2). The most expedient proportions in terms of energy reception have buildings of corridor and gallery type, which have been shown in the course of analysis of the correlations between techno-economical, physical and technical indicators of buildings.

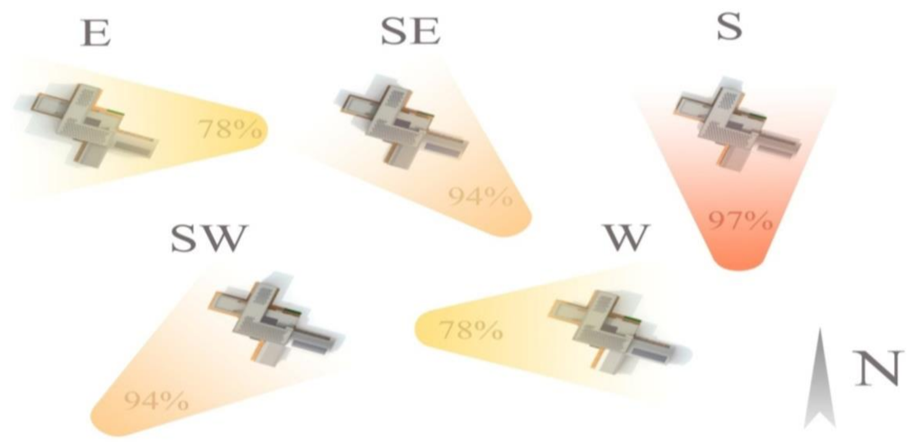


Fig. 2. Evaluation of energy efficiency of buildings orientation, the resort-and-health-improving complex

The research that has been carried out and the experience of power equipment development show that the integrated solar systems shall be considered as a component of entire power supply system to make most effective use of them. That is why an integrative approach is needed to study and to take into account all aspects including buildings' energy characteristics and regional climatic conditions, specific features of engineering systems and household appliances in order to minimize losses and to increase the energy supply to the end user [3, 4].

Solar modules and collectors are integrated into the cladding structures and are mounted on the roofs of the main buildings (Fig. 3). The project includes passive solar constructions - solar trap (Fig. 3, 4).

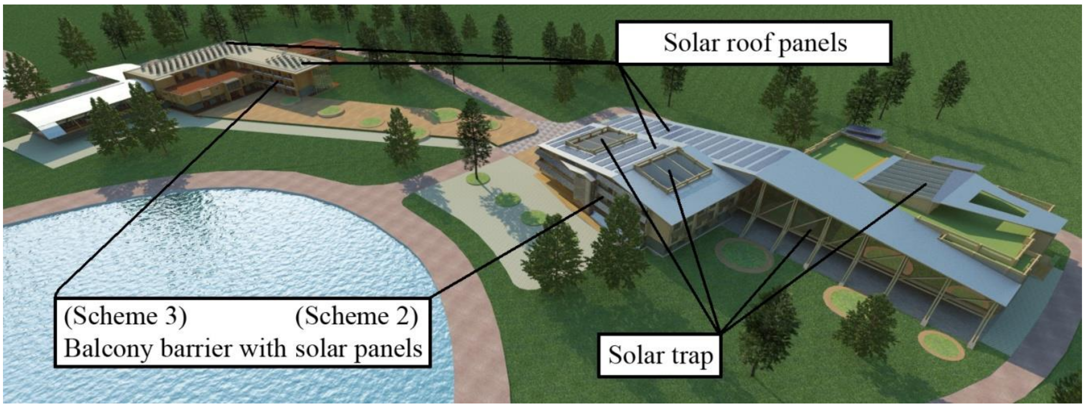


Fig. 3. Resort-and-health-improving and educational-and-recreational complex with BIPV

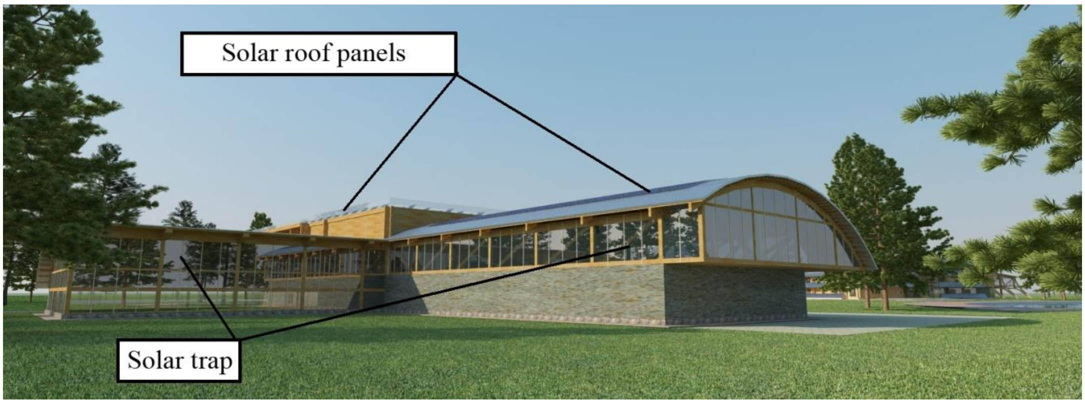


Fig. 4. Passive solar constructions, the resort-and-health-improving complex

The balcony barriers were developed on the basis of three general module designs of balcony barrier. The constructive diagrams of module for balcony barrier are represented in Figure 5.

The first scheme shows a balcony barrier of screen type with filling of its framework. The screens are arranged in form of solar panels (Fig. 5a). The second scheme consists of metal racks and console attached to them solar panels (Fig. 5b and Fig. 3, 6b). The third scheme is a barrier frame which permeates panels and solar modules mounted on frame so that their tilt angle can be adjusted (Fig. 5c).

For usage of integrated solar panels as balcony barriers in Russia their extent shall be devisable by 300 mm or 100 mm. Balcony barrier standard height shall comply with GOST 25772-83: 1000 mm for buildings lower than 30 m; 1100 mm for buildings in higher than 30m; 1200 mm for preschool institutions; 1200 mm for smoke controlled staircases.

The third general design (Fig. 5a) was chosen in the course of developing the sketches of building exteriors (see Fig. 3) so that the solar panels in the module of balcony barrier could be turned around the horizontal axis since, depending on the latitude, panels shall be installed at a specific angle to the horizon. Besides, modules of shorter length can performed on the basis of the third scheme type having a vertical rotation axis for application on building facades that are not South-oriented.

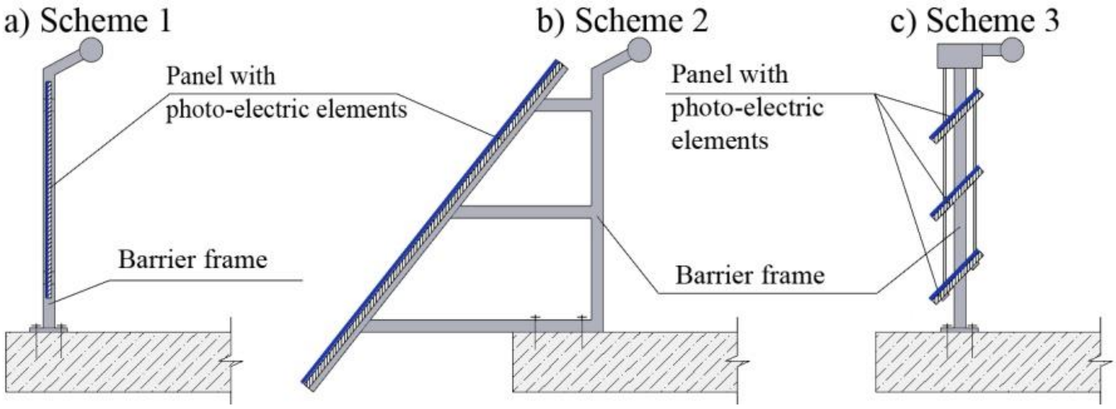
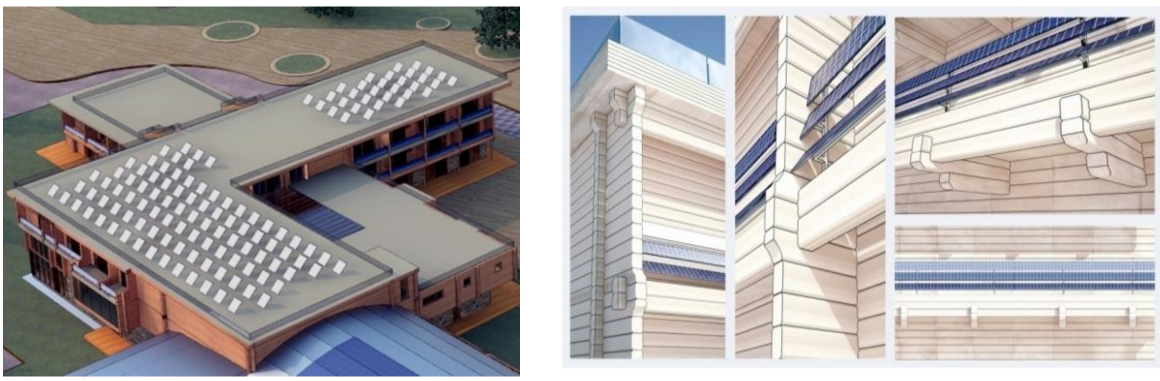


Fig. 5. General constructive diagrams of the module of balcony barrier



a **b**

Fig. 6. Fragments of RHIERC with BIPV: a - roof fragment; b - facade fragment

While designing the mechanical structure of balcony barrier modules with solar panels one shall consider the possible wind load rates and mechanical durability of both the balcony barrier and the solar panels. Therefore, taking into consideration the maximal sizes of solar modules and collectors, the linear sizes of integrated panels and those of balcony barrier shall be corrected to reduce the external dynamic loadings without reduction of total light receiving area of panels.

In the project, it is intended to explore the difference of modules and collectors in size for organizing the facade exterior, while the difference in their colour will be taken into account to create specific colour solutions of facades.

The exterior of RHIERC buildings with approximately uniform energy demand during the year for projects designed for Central Russia is shown in Figures 6a, 6b taking the seasonal variations into account. On the average, for a sunny July day solar panels on the basis of planar PV cells having the capacity of 200 W/m^2 generates 800 to 900 W/m^2 a day. With respect to the shading and architectural solutions the optimal angles solar panels of inclination shall be from 36 to 65…70 degrees.

The power generation potential dependence on weather conditions shall be used while planning the solutions for buildings and adjacent territories, by the selection of power equipment combinations and their connection combinations, accumulation options and that of energy saving loads management organization.

To determine the efficiency and to choose an optimal planning solution it is necessary to carry out the calculations based not only on the aggregate annual solar radiation for particular area but also on the basis of seasonal changes. In most cases it is sufficient to take the summer-to-winter ratio into account. If the load is not reduced during the winter the big difference between winter and summer solar radiation makes the system design and functioning more complicated. If the case that the system has been winter-sized in respect to the maximum energy demand then the large surplus power generated in the summer has to be somehow accumulated for use in the winter period which is rather problematic today, while its lending or redistribution is not always possible. If the system has been sized on the basis of average annual or summer radiation rate the proper sizing of batteries and connections can only partially compensate for the lack of energy deficit. Therefore, the use of an additional or second main power source is required for the winter period. For example, in Middle Russia the electric power winter-to-summer ratio is 8/1. Sevastopol where summer radiation rate is quite high but the summer/winter ratio is even worse than for Middle Russia is less acceptable for an autonomous RHIERC with year-round power supply. At the same time, Primorye where summer power generation rates are lower than for Moscow but the winter-to-summer ratio is nearly unity solar systems are more advantageous to be installed.

The architecture of buildings is formed by combination of integrated solar panels with energy effective building solutions using natural materials. With the vertical solar panels installation the amount of excess energy in the summer decreases while the operation efficiency in the winter time is optimal owing to the sunlight falling at a smaller angle. For latitude 58 degrees, the integrated stationary photovoltaic equipment installed vertically (at an angle of 90 degrees) or with a tilt angle changing 2 times per year (March-November 36 to 50 degrees, November-March 90 degrees) is the most effective.

Structures were designed for solar modules of two types, for modules with conventional gradient-type cells and with multijunction planar cells [5, 6]. Fig. 7 shows diagram of system with planar PV cells. According to calculations, the second type provides higher efficiency and simpler circuitry design solutions. The outer structures of module are supposed to be uniform therefore the exterior of buildings remains unchanged. Though the second variant is preferable in terms of orientation it has some particular features concerning the entire organization system which requires a certain correction of service premises planning.

Table 1 shows result of design experiment. Total PV-power is more than 0,45MWatt, total estimated production with only PV modules is more than 542 MWatt hours per year.

To enhance the efficiency the glass coating of solar collectors consists of two connected glasses with the gap of 150μ between them evacuated to 5 x 10^{-5} mm Hg. The similar concept is applied to the building external envelope which makes it possible to reduce substantially the heat losses and, accordingly, to increase the share of solar systems in the energy balance of buildings and to use passive solar constructions effectively (Fig. 8, 9).

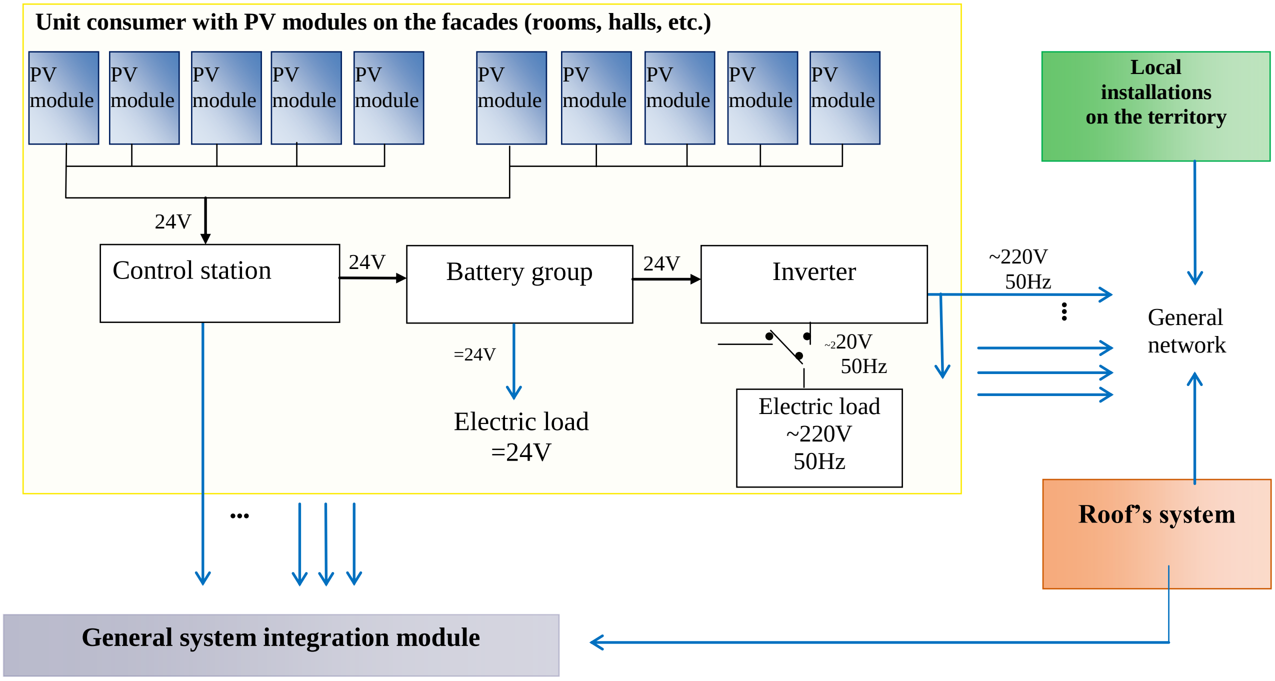


Fig. 7. Diagram of system planar PV cells

Table 1. Result of RHIERC design experiment

	Position	**Orien-tation**	**Position area (sq.m)**	**PV-power (kWp), min**	**Estimated production (kWh/year), only PV modules**	**Estimated production (kWh/year), 30% collectors**
Education-al-and-recreational complex	Roof	S	1462	153,510	189431,34	132601,938
	Balcony barrier	SE	281	42,150	48379,77	33865,839
		SW	141	21,150	23406	16384,2
Resort-and-health-improving complex	Roof	S	1403	147,315	181786,71	127250,697
		SW	180	27,0	28398,6	28398,6
	Balcony barrier	SE	90	13,5	15495,3	10846,71
		SW	225	33,750	37350	26145
		W	135	20,250	18589,5	18589,5
Total:				458,625	542837,2	394082,5

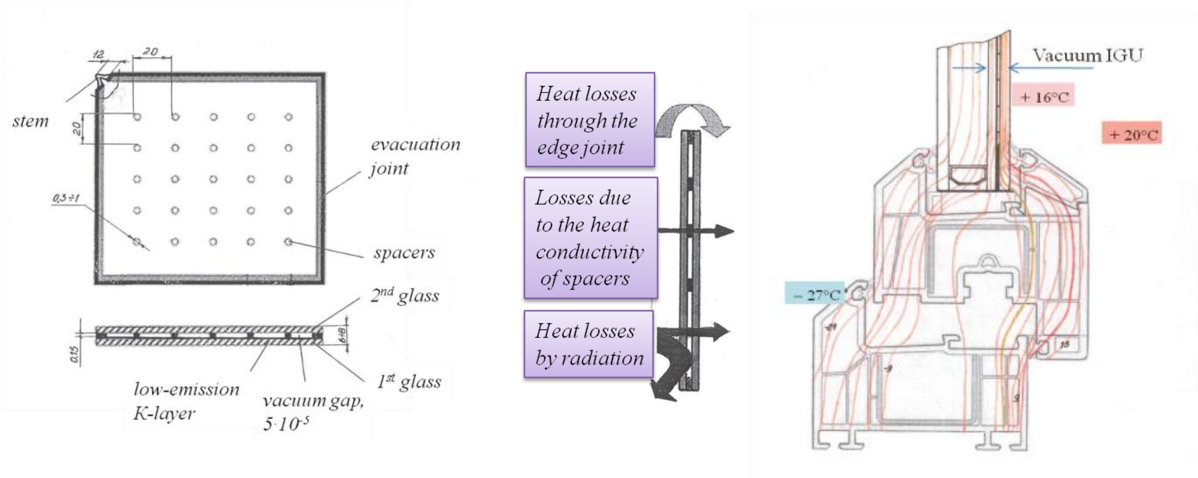
The novelty of this work as a "Green Project" is in developing of:

1) Energy-independent buildings made of environmentally benign materials (wood, stone) and establishing the functional correlation between rehabilitation and education;

2) Principles of town-planning design of energy-independent RHIERCs in accessibility areas of objects of historical and cultural heritage, areas under strict protection and environmentally benign rural settlements;

3) Development of sustainable architectural, constructive, and space-planning decisions and compositional structure of RHIERC buildings equipped with the use of renewable energy technologies.

The project promotes the development of renewable energy sources technology, classified by Russian Government as crucial, in more advantageous conditions in terms of economy. This will make it possible for small and medium enterprises to participate in investments into renewable energy technologies used in buildings architecture. In addition, the problem of aesthetically appealing buildings' appearance made of eco-friendly materials and equipped using renewable energy technologies can be successfully solved.



a) Design **b) Heat losses** **c) Base variant isotherm distribution**

Fig. 8. Vacuum insulating glass unit (IGU)

Fig. 9. Building external envelope with Vacuum IGU

The results of research made it possible to incorporate into one integrated method the evaluation of buildings' energy potential followed by the architecturally-constructive design [7]. The energy balance are determined from the comparison of energy generation rate by the equipment is integrated into the building and the amount of energy required for its normal operation. Where an energy deficiency is obtained the decision can be corrected for the building's perimeter extension and increasing the total area of solar panels generating electric power. In case that the energy is generated in excess to demand the decision can be made by designers to extend the total housing area of building.Thus incorporation of the power independent buildings and installations having exterior-integrated solar equipment into the RHIERC concept can provide the basic energy needs of complexes and most part of ecovillages' energy supply.The original method of energy potential evaluation for territories and buildings for architectural and construction design with RHIERC as an example presented by the authors can be perceptively used for objects having other functions.

Conclusion

The innovation potential of RHIERC research project in ecovillage GENOM is based on the results application for environmentally oriented development of small and medium business in resort-recreation sector, as well as for the reorientation of rural enterprises or providing them with agro-tourism functions aimed at the increasing their financial potential on the basis of proper management of national resources.

The present project will demonstrates that power-independent, aesthetically and economically appealing RHIERC and ecovillages can be implemented with the primary energy supply by solar energy.

The practical importance of the project is the creation of convenient conditions for medical treatment, rehabilitation, education and environmentally oriented training of tourists and holidaymakers, as well as for the development of ecovillages infrastructure in the housing planning of rural areas.

The timeliness of results is based on the scientific-methodical extension and project substantiation of development in the sphere of three-dimensional-spatial and typological solutions for power independent buildings using RHIERC as an example.

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INNOVATIVE ELECTRICAL ADJUSTING PRODUCTS IN LOW-VOLTAGE ELECTRICAL INSTALLATIONS AND NETWORKS

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Grounding of cases and conducting covers of the low-voltage equipment is an effective remedy of decrease in intensity of electromagnetic radiation of electrical installations and electric devices in close proximity to the person. The innovative electric adjusting products for single-phase two-wire power supply networks allowing without reconstruction of these networks are considered it is reliable and safe to ground low-voltage electrical installations.

Keywords: *electric fields, magnetic fields, electrical installations, electrical networks, grounding, electrical adjusting products, fork, socket*

The electromagnetic field of the earth, radio emission of the sun, galaxies and atmospheric categories belong to sources of natural electromagnetic fields. Being constantly operating ecological factors, this field has special value in activity of all organisms living on the earth including the person [1].

Resultant level of a natural electromagnetic field is much lower than levels of electromagnetic fields created by artificial (ethnogeny) sources therefore more and more increasing interest in a problem of impact of the last on health of the person is connected with expansion of scope of the electric power for various technological processes in production and daily life.

Sources of artificial electromagnetic fields are electrotechnical, radiotechnical, television objects, household equipment. Electromagnetic fields emitted by artificial sources, together with the natural electromagnetic fields form a total intensity of EMF at some points of the earth's surface is several hundred times more of intensity of natural EMF [1].

The World Health Organization (WHO) entered the term "electromagnetic pollution of the environment", which reflects the new environmental conditions prevailing on the ground in terms of EMF exposure on humans and other elements of the biosphere.

By results of numerous medicobiological researches in the field of impact of EMF on living organisms intensively conducted in many countries it is possible to state that:

1. EMF have biological activity in all frequency bands. Two mechanisms of impact of EMF on the person are installed: excitement of nervous cells of fabrics the currents induced in them by external EMF, and heating of fabrics of an organism due to absorption of energy of a field [2] by them. Cells of nervous and muscular fabrics are most sensitive to excitement in the range of frequencies from 10 to 1000 Hz. With increase in frequency of the influencing field sensitivity to excitement falls and influence of a field is shown in the form of fabric heating.

2. As reactions of the person to influence of EMF are revealed such as adaptation and a cumulativeness. Adaptation is expressed that with growth of level of influence of EMF reaction of an organism at first increases (within threshold value), but then falls as various compensation actions of an organism join, that is the person gets used to action of EMF and repeated influences practically don't influence its state.

The cumulativeness is shown in possibility of accumulation of biological effect of EMF in the conditions of their long years of exposure.

3. Reaction of a live organism to influence of EMF has pronounced individual character. The person feels influence of EMF on vibration of hair because of existence on them of a charge or in the form of an itch of some parts of a body. Approximately 80% of people are beginning to feel the electric field strength of 80 kV/m, and 5% - 7 kV/m [3].

The magnitude (intensity) of an electromagnetic field is characterized by such parameters as the intensity of the electric field (EF) denoted by E and measured in volts on meter (V/m) and intensity of the magnetic field (MF), denoted by H and measured in amperes on meter (A/m). The magnetic field is characterized often by magnetic induction of B measured in teslas (T).

To ensure the safety of people in the zone of influence of electromagnetic fields are developed norms of admissible impact of EMF on the person based on medicobiological researches.

At action of EMF both the electric, and magnetic component of a field in this connection EMP as on an electric, and magnetic component of a field is normalized has impact on the person on it.

In our country maximum permissible levels of intensity of electric field with a frequency of 50 Hz for the personnel serving electrical installations and being in a zone of the influence created by them EP depending on time of stay in EP, and also the requirement to monitoring procedure of levels of intensity of EP on workplaces are established [4].

Maximum permissible level of intensity of the influencing EF at stay in it during the working day established 5 kV/m inclusive.

Admissible time of stay in EF intensity from 5 to 20 kV/m inclusive is calculated by the formula:

$$\tau = \frac{50}{E} - 2,$$

where τ – admissible time of stay in EF at the level of intensity of E kV/m.

At intensity from 20 to 25 kV/m time of stay of the personnel in EF should not exceed 10 min.

Stay in EF intensity more than 25 kV/m without application of means of protection is not allowed.

If necessary to define maximum permissible intensity of EF at the set time of stay in it use a formula:

$$E = \frac{50}{\tau + 2},$$

where τ– the set time of stay in EF.

The magnetic field and currents induced by it at hours-long influence with intensity H=1600 A/m didn't cause any deviations in health of people, strong spasms come only at the intensity higher than 105 A/m [3]. Admissible intensity of a magnetic field is established depending on time of stay in it [5].

At stay time within 8 hours are established by admissible values of a magnetic field H=80 A/m and B=100 μT, at the general influence are determined less than 1 hour by admissible values of MF H=1600 A/m and B=2000 μT.

Grounding of cases and conducting covers of the low-voltage equipment is an effective remedy of decrease in intensity of electromagnetic radiation of electrical installations and electric devices (the electric tool, office equipment, refrigerators, computers, irons, electric kettles, electric furnaces, washing machines, vacuum cleaners, etc.) in close proximity to the person.

In production and premises of considerable part of the operated buildings and constructions two-wire single-phase electric networks are applied. In such networks performance of grounding of electrical installations and electric equipment is problematic, demands laying of the additional grounding protective conductor that is connected with the consumption of materials and considerable labor costs.

The new technical solution allowing to ground reliably and safely electrical installations without reconstruction of low-voltage networks in which there are no the grounding PE-conductors is proposed. Innovative electrical adjusting products for single-phase two-wire power supply networks include plug forks and three-contact plug receptacles of the european type with the grounding element in cases of monolithic execution [6 - 9].

In the offered two-pin monolithic plug fork with the grounding element in the case there is a conducting restriction (limiting) element which one end is attached to the fork probe which is in turn connected to a zero working wire of an electric network, and the second end of an element serves for accession to it of the grounding conductor of a three-vein electrical cord of an electrical household appliance, the electric tool. The tiny neon bulb which one conclusion is attached to the metal contact come on the fork case, and the second by the end of the restriction element attached to the grounding electric device electrical cord vein is built in the case of a fork. The neon bulb is intended for the alarm system of connection of the probe of a fork and a restriction element with a zero working wire on lack of a luminescence of a bulb in an opening of the case of a plug fork at a touch a hand finger to metal contact. The cavity for placement in it the replaced tiny neon bulb is executed with an opening outside and two reliable electric contacts for food of a bulb (fig. 1).

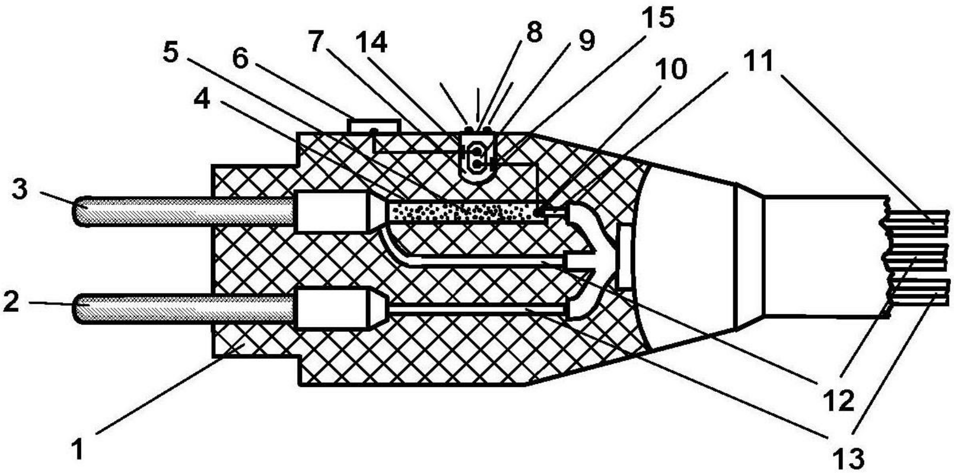

Fig.1. Two-pin plug fork with a grounding element in a monolithic case:

1 – the monolithic non-demountable case of a fork; 2 – the fixed metal pin for connection with an electric network; 3 – the fixed metal pin for connection with an electric network; 4 – hidden cylindrical cavity; 5 – conducting material; 6 – metal contact; 7 – a cavity in the case for containing a neon bulb; 8 – the opening in the case; 9 – miniature neon bulb; 10 – the grounding contact for fastening of the grounding conductor; 11 – the grounding conductor of an electrical cord of the electric device; 12 – the zero working conductor of an electrical cord of the electric device; 13 – the phase conductor of an electrical cord of the electric device

The monolithic non-demountable plug fork works with a restriction element as follows. The fork is inserted into the ordinary two-contact socket, and then touch with a finger of a hand metal contact 6 on the case of 1 fork and through an opening 8 in the case 1 define existence or lack of a luminescence of a neon bulb 9. Existence of a luminescence of a bulb 9 testifies to the wrong connection of a fork. The fork should be established anew, having turned on 180 degrees, i.e. to trade places probes 2 and 3. Thus the neon bulb shouldn't shine that will testify to the correct grounding of the electric device through the grounding conductor of the 11th electrical cord of the electric device. The replaced neon bulb 9 placed in a cavity 7 with an opening outside 8 and electric contacts 14 and 15 for connection of a neon bulb in the case of 1 fork can be replaced at its malfunction or mechanical destruction. The electrical cord of the electric device includes three conductors – the grounding conductor 11, the zero working conductor 12 and the phase conductor 13. The restriction element in the form of the hidden cavity 4 in the case of 1 fork which is filled with conducting material 5, is the limiting element in a chain of grounding of an electrical household appliance and limits current through a neon bulb 7 at a touch to a hand finger to metal contact 6 on the fork case.

In the developed three-contact plug receptacle of the european type with the grounding element representing a usual three-contact plug receptacle of the european type with the case from insulating material, two metal nests – the restriction element (resistor) which one end is connected to zero working contact, and the second – with grounding is the phase and zero worker and the double grounding contact, in the case.

The restriction element is executed in the form of the hidden conducting cavity in the monolithic case of a three-contact plug receptacle, thus one end of a cavity is attached to a socket nest which is connected to a zero working wire of an electric network, and the second is attached to one of doubled (to one of two, connected among themselves) the grounding contacts of the socket intended for connection with the grounding conductor of a three-vein electrical cord of the electric device or low-voltage electrical installation.

The tiny neon bulb which one conclusion is attached to the metal contact come on the socket case, and the hidden conducting cavity, the second by the end attached to the grounding contact of the socket is built in the case of the socket. The bulb is intended for the alarm system of connection of a nest of the socket and a restriction element with a zero working wire on lack of a luminescence of a neon bulb in a through opening of the case of a plug receptacle at a touch a hand finger to metal contact on the case of the socket (fig. 2).

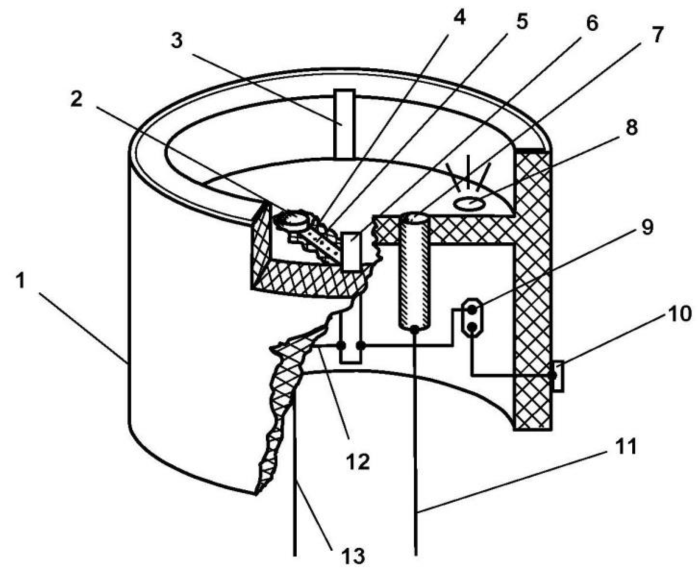

Fig. 2 Three-contact plug socket the european type with the case of monolithic execution:

1 – the monolithic case of the socket; 2 – the metal nest fixed in the case for connection with an electric network; 3 – the metal grounding contact; 4 – hidden cylindrical cavity; 5 – conducting material; 6 – the metal grounding contact; 7 – the metal nest fixed in the case for connection with an electric network; 8 – the through opening in the socket case; 9 – miniature neon bulb; 10 – metal contact; 11 – the phase conductor of an electric network; 12 – the zero working conductor of an electric network; 13 – the conductor connecting the grounding contacts 3 and 6

The three-contact plug receptacle works with a restriction element as follows. The socket is installed in the ordinary two-wire feeding power supply network, and then touch with a finger of a hand metal contact 10 on the case of 1 socket and through a through opening 8 in the case 1 define existence or lack of a luminescence of a tiny neon bulb 9. Existence of a luminescence of a bulb 9 testifies to the wrong connection of the socket. The socket should be installed anew, having traded places the attached power supply network wires. Thus the tiny neon bulb shouldn't shine that will testify to the correct accession of the zero working conductor 13 and to the subsequent reliable grounding of the electric device or electrical installation through the connected three-wire electrical cord of these of the electric device or electrical installation. The electrical cord with a plug fork of the European type connected to the ordinary two-wire feeding power supply network with the offered socket contains three conductors – the grounding conductor connected to contacts 3 and 6, the zero working conductor connected to a nest 2 and the phase conductor connected to a nest 7. The restriction element which is in the case of 1 socket in the form of the hidden cylindrical cavity 4 which is filled with conducting material 4, limits current through a tiny neon bulb 9 at a touch to a hand finger to metal contact 10 on the case of the socket 1 and is the limiting element in an electrical installation grounding chain.

The restriction element applied in developed to a fork and the socket is carried out in the form of the hidden extended cylindrical cavity with a diameter not less than 2 mm in the monolithic case which is filled with conducting material, for example, conducting plastic, plastic or other elastic not fragile conducting material with a cavity volume resistance from 10 to 100 kΩ. Diameter of a cylindrical cavity not less than 2 mm provides necessary conductivity and mechanical stability if deliberate destruction of the case with total loss of functional mission of products isn't allowed.

Data on the products which are thought over by innovative electrical adjusting can be placed in the formalized look in knowledge bases of systems of electronic training of the personnel on electrical safety and network systems of support of decision-making on reduction in production electrical injuries and creation of electrical safety of working conditions [10…13].

Conclusions

1. Application of the developed bipolar fork and a three-contact plug receptacle of the european type with the grounding element in the non-demountable monolithic case excludes access to operation time to the electric contacts located in the case and to the hidden conducting area which is carrying out a role of the restriction (limiting) element.

2. The developed innovative electrical adjusting products of the offered design provide reliable and safe grounding of electric equipment and electrical installations in two-wire low-voltage networks, provide decrease in level of impact of electromagnetic fields on the person from the working electrical installations and electric equipment and allow to automate completely their production with decrease in number of technological operations in an industrial cycle of production of the modified adjusting products.

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BIOGAS TECHNOLOGY REVENUE POTENTIAL OF AGRICULTURE IN THE TAMBOV REGION OF THE RUSSIAN FEDERATION

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In this study the biogas technology revenue potential is estimated for the Tambov region of the Russian Federation in general as well as for its municipal districts in accordance to standard biogas, electricity, heat and NPK outputs for each substrate. Biogas potential evaluation for the Tambov region of the Russian Federation in this article considers calculation methodology, which can be applied afterwards for the whole country.

Keywords: *biogas, electricity, heat, NPK, revenue, potential.*

Introduction

Biogas potential of agriculture in Tambov region is closely related to its main agricultural production (crops, animal and poultry wastes) on the one hand and to technical availability and economic validity on another [1], [2], [3]. Therefore it is assumed to identify the following substrates to take into account as the feedstock for biogas technology using potential estimation: cattle and pig manure and poultry dung, maize and grass silo, cereal grain of winter wheat, winter rye, winter triticale, spring wheat, spring barley, oat, millet and sugar beet.

Research method

Estimation of biogas technology using potential is the widespread approach. Closely related to each other elements of this methodology are used in many countries [4], [5], [6], [7], [8], [9], [10].

$$R = R_E + R_H + R_F$$

where: R – revenue potential from using biogas technology, rubles; R_e – revenue from electricity production, rubles; R_h – revenue from heat production, rubles; R_f – revenue from fertilizers production, rubles.

$$R_E = EP \times p_e$$

where: EP – electricity potential, kWh; p_e – electricity price, rubles/kWh.

$$R_H = HP \times p_h$$

where: HP – heat potential, Gcal; p_e – heat price, rubles/Gcal.

$$R_F = FP \times p_f$$

where: FP – fertilizer potential, t; p_f – fertilizer price, rubles/t.

$$p_f = q_N p_N + q_P p_P + q_K p_K$$

where: q_N – nitrogen content per t of substrate, t; q_P – phosphorus content per t of substrate, t; q_K – potassium content per t of substrate, t; p_N – nitrogen price rubles/t; p_P – phosphorus price, rubles/t; p_K – potassium price, rubles/t.

$$p_{AM} = \frac{p_f^{AM}}{AM_{share}}$$

where: p_{AM}- fertilizer active material (N, P_2O_5 or K_2O) price, rubles/t; p_f^{AM}- single fertilizer price, rubles/t; AM_{share}- active material share.

$$EP = BP \times E_{output}$$

where: BP – biogas potential per unit of substrate, m^3/t; E_{output} – energy output per biogas unit, kWh/m^3. The average energy output per 1 $m^3$of biogas is assumed to be equal to 2 kWh/m^3 and to be constant [11].

$$HP = BP \times H_{output}$$

where: H_{output} – heat output per biogas unit, Gcal/m^3.

$$H_{output} = E_{output} \times 0{,}001032$$

where coefficient 1,032=2×0,4×0,00086 reflects the statement that experience shows approximately 2/3 of energy produced from biogas is the heat, nearly 40 % of which is used for biogas production process [11] and measured by the FSSS of FR in Gcal [2].

$$BP = Q_{resource} \times S_{output} \times BG_{output}$$

where: $Q_{resource}$ – resource amount, animals or t of crop; S_{output} – substrate output per unit of resource, t/animal or t/t; BG_{output} – biogas output per substrate unit, m^3/t.

Overall regional revenue potentials of electricity, heat and fertilizers $(R_E^R;\ R_H^R;\ R_F^R)$ are calculated taking in account all kinds of resources and all standard kinds of substrates that can be used in agriculture and processing production for biogas production.

Therefore, there overall biogas revenue potential of the region is calculated with help of formulae:

$$R^R = R_E^R + R_H^R + R_F^R$$

$$R_E^R = p_e \times \sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n} EP_{ijk}$$

$$R_H^R = p_H \times \sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n} HP_{ijk}$$

$$R_F^R = p_N \times \sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n} NP_{ijk} + p_P \times \sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n} PP_{ijk} + p_K \times \sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n} KP_{ijk}$$

where: NP – nitrogen potential, t; PP – phosphorus potential, t; KP – potassium potential, t; i – substrate index; j – resource index; k – district index; n – available substrates amount; m – available resources amount; l – districts amount.

Experimental part

The table 1 [3], [4], [11], [12], [13], [14], [15] shows substrates characteristics. Mineral fertilizers parameters, energy prices as well as biogas resources (animals, poultry and crops) in the Tambov region of the Russian Federation in 2012 are presented in data bases of Federal State Statistical Service of Russian the Federation [2].

Table 1

Biogas substrates characteristics

Parameter	Cattle manure	Pig manure	Poultry dung	Maize silo	Grass silo	Grain
Annual substrate output per unit of resourse, t/animal or t/t of production	10,95	1,46	0,073	0,75	0,75	1
Biogas output per unit of substrate, m^3/t	80	60	140	200	180	620
CH_4 output, m^3/t	44	36	90	106	98	329
CO_2 output, m^3/t	36	24	50	94	82	291
Biogas output per unit of substrate, t/t	0,10	0,07	0,16	0,26	0,23	0,81
Digestate output per unit of substrate, t/t	0,90	0,93	0,84	0,74	0,77	0,19
Dry matter, % FM	25	22,5	40	33	35	87
N-output, % DM	5,6	2,3	18,4	2,8	4,0	12,5
P_2O_5-output, % DM	3,2	3,1	14,3	1,8	2,2	7,2
K_2O-output,% DM	8,8	1,8	13,5	4,3	8,9	5,7
N-output, % FM	1,40	0,57	4,60	0,70	1,00	3,13
P_2O_5-output, % FM	0,80	0,78	3,58	0,45	0,55	1,80
K_2O-output,% FM	2,20	0,45	3,38	1,08	2,23	1,43

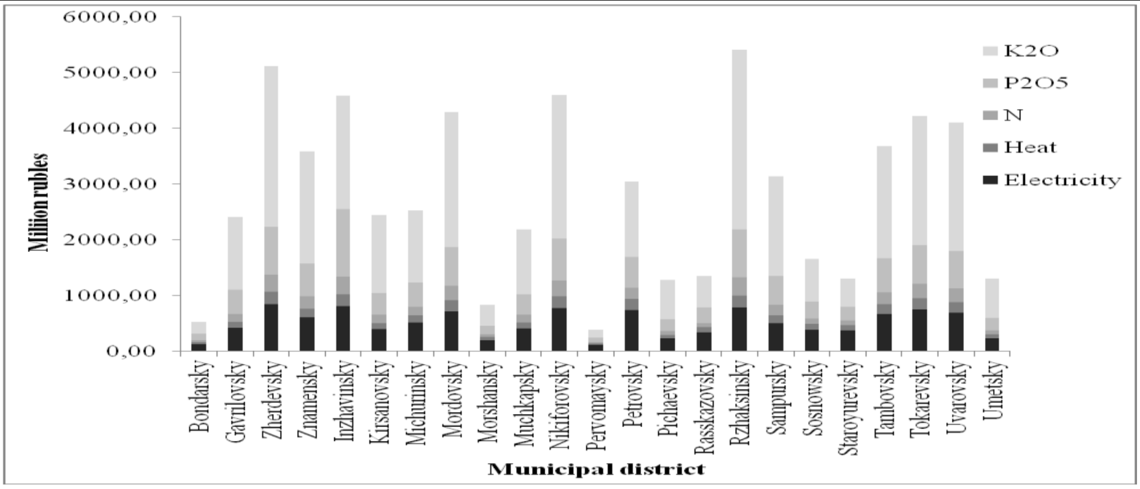

Fig. 1. Revenue potential arrangement between substrates and municipal districts

Results

The overall revenue is estimated to be equal to about 64 billion rubles. The biggest share of it is taken by digestate (76,9 %), where K_2O, P_2O_5 and N cover 53,2; 17,6 and 6,1 % respectively. Electricity covers 18,3 % of overall revenue potential, while heat is on the last place and takes 4,8 %.

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MODERN ADAPTATION APPROACH OF AGRICULTURE TO CLIMATE CHANGE AND RESERVOIRS IMPACT

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This article discusses the impact of Nurek reservoir and climate change on agricultural sector in Fayzabad and Dangara districts, which have been observed during 1950-2012 by meteorological stations. Also the article analyzes changing meteorological parameters (temperature, precipitation, humidity, evaporation) and adaptation of agricultural sector to the process of global climate change as well as the models of their mitigation.

Keywords*: reservoir; agrolimatic condition; adaptation; humidity; evaporation; agriculture; mountain; irrigation*

Introduction

Hydropower equally with agriculture is the basic economical branch of the Republic of Tajikistan. The total annual of hydropower resources of the Tajikistan is 527 Bln. kWt·h that now used only 5 %. The presence of the rich potential of the production of electricity in Tajikistan suggests that in the short term expected to build a number of small and large hydroelectric power plants with reservoirs [1].

Therefore, at planning prospects for agriculture coastal areas to the reservoirs must take into account the influence of water reservoirs in the transformation of the meteorological conditions of the area and make correction to the irrigation norm of the relevant agriculture crops.

The aim of the present paper is a retrospective comparative analysis of statistical parameters of 60 - year time series of temperature, humidity and monitoring of the Nurek reservoir influences on trend changes of these parameters.

Methodology

To determine the influence of mountain reservoirs on agroclimatic conditions we analyzed the trend of meteorological parameters the Dangara district of Tajikistan with developed agriculture that is a coastal to the Nurek reservoir. We used meteorological data 1950-2012 years from station located in the study area. Nurek dam construction was started in 1961 year. The water level at 1979 has been reached 890 m and the normal water level equal to 910 m was achieved in September 1983. Therefore, we can assume that the influence of the reservoir on meteorological parameters area should be observed after 1980 years.

Based on this assumption, we analyzed meteorological parameters of the two periods - before (1950-1980) and after (1981-2012) the construction of the dams.

Results and discussion

Temperature change in the Dangara district for the period 1950-2010 years characterized by its uniform increase without any extreme evidences about influence of the Nurek reservoir (Fig. 1).

Natural to expect that the manifestation of any signs of the influence of the reservoir on the temperature variation due to smoothing them for such a long period is very difficult. For a more detailed study of the influence of the reservoir on the average temperature, we carried out separately systematization of meteorological parameters of Dangara district before and after the construction of the reservoir. Taking into account that the weather of Tajikistan is continental, we considered the trend of temperature change in winter and summer seasons of the considering periods.

Annual average temperature change in winter (XI-II) (a) and summer (V-VIII) (b) before and after construction of the Nurek reservoir shown in Fig. 2 and 3.

The data on Fig. 2, 3 show that before 1980 when water level was not reached Normal water level observed increase in temperature occurs mainly in summer. After filling the reservoir by water to full mark (after 1980), the picture will change to the opposite, i.e. to increase the temperature in the winter.

The obtained results give reason to believe that the reservoir acts as a conditioner weather conditions in the settlement areas.

Analysis of annual precipitation shows that for the period 1960-2010 the cyclical fluctuations by interval of 3-5 years were observed (Fig.4).

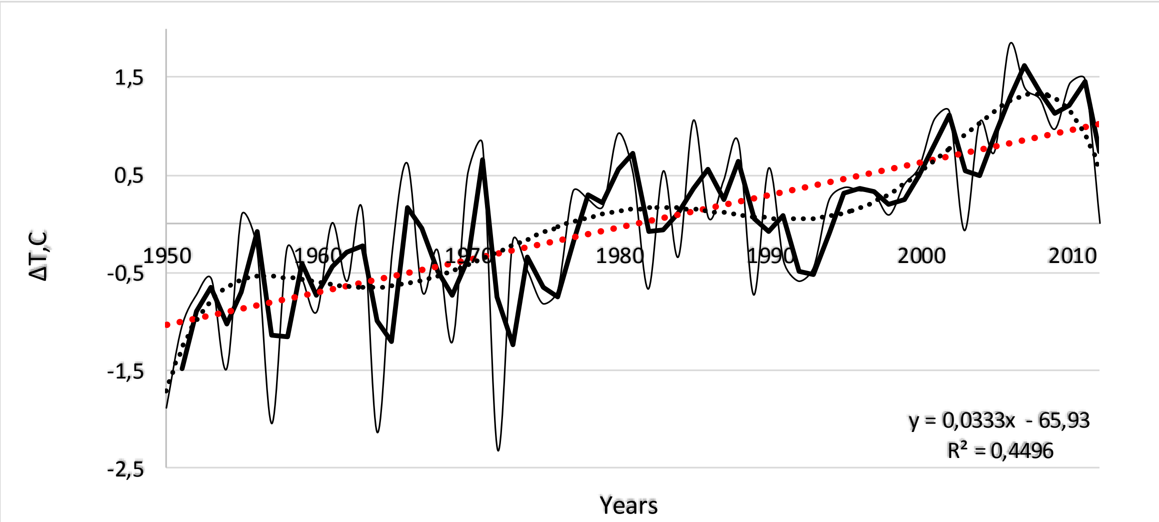
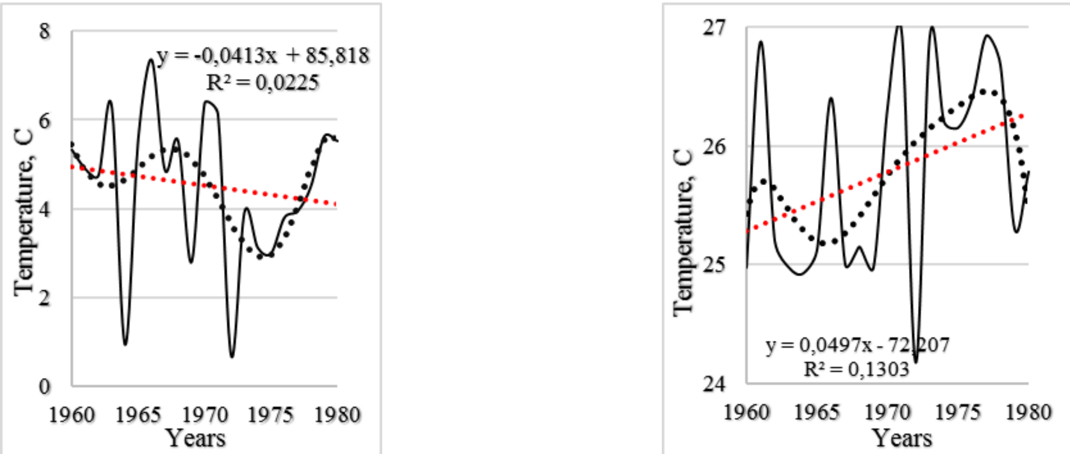
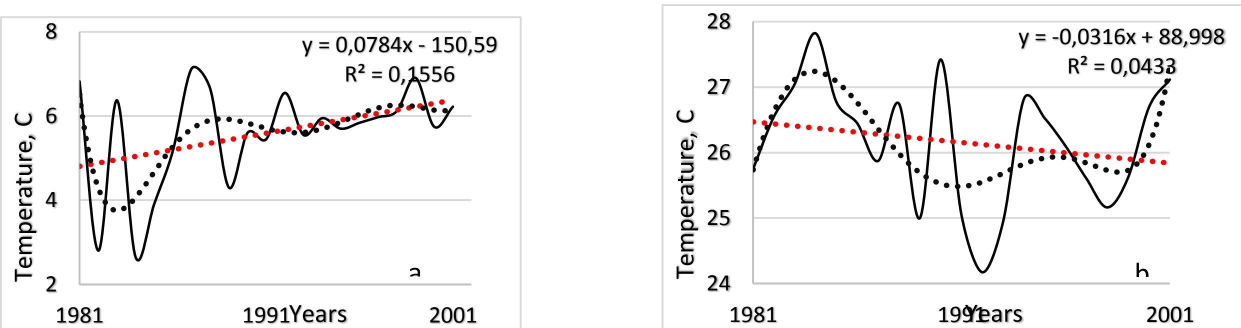


Fig.1. Long-term course of mean annual temperature deviations from the average values for the period 1950-2012 years



a *b*

Fig.2. Long-term average annual temperature of Dangara in winter (a) and the summer (b) for the period of 1960-1980 years



a ***b***

Fig.3. Long-term average annual temperature of Dangara in winter (a) and the summer (b) for the period of 1981-2001 years

Humidity reduction for the considered periods that is connected with increase of temperature at almost invariable trend of the precipitation change in Dangara was observed (Fig. 5).

It is established that in the conditions of Global climate change and its influence on all components of an ecosystem to become actual a problem of development of adequate and modern methods of adaptation of human activity to cataclysms of climate. In agriculture first of all substantial increase of efficiency of irrigation water and a farmland and wide involvement of biotechnology for selection high - efficiency and steady against stressful situations of grades is necessary. In the hydropower production direction this effective placement of hydropower station with reservoirs and stability of dams.

For establishment influences of the climate change on possible changes of agroclimatic resources we were spent the analysis of climatic parameters of three districts with developed agricultural branches (Dangara, Fayzabad and Yavan) adjoined to the Nurek reservoir. For this purpose, data of Hydrometeorological stations located in these areas have been used. For 45 years (1968-2013), the average annual temperature has raised on 1.0-1.5°C that has led to decrease of the relative humidity on 3-6% and to increase evaporation on 10-26 % in an annual cut and 12-30 % in period May- September. However, in Yavan district dynamics of changes of the listed parameters has the opposite tendency: the temperature of air and evaporation decreases accordingly on 0.5, 7.2 %, relative humidity, and factor of humidifying raise on 7.2 % and 10 % accordingly.

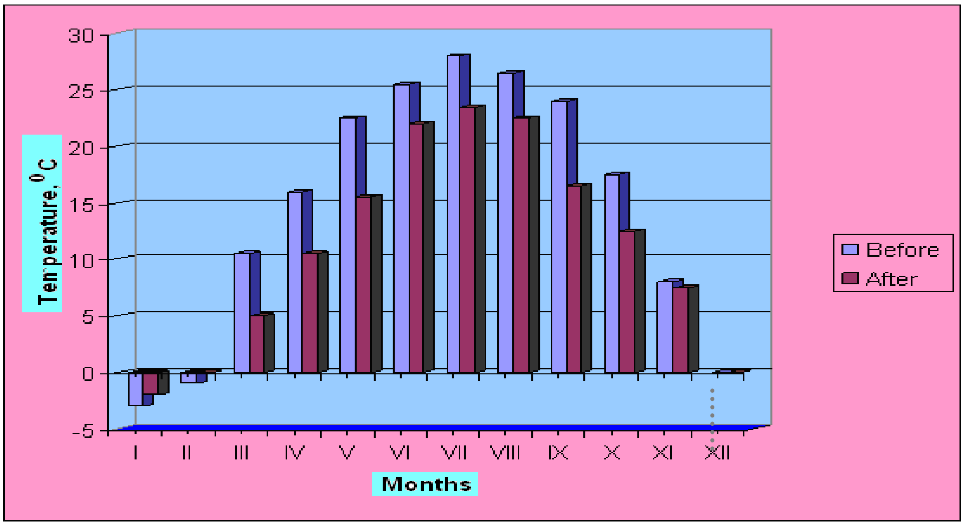


Fig.6. Mid-monthly temperature before and after building of the Nurek HPS

In view of climatic changes, it is necessary to bring corresponding corrective amendments in planning of the water use in agriculture. At development of regime of the irrigation, it is usually considered parameters of meteorological condition for all period of supervision. However, it conducts to essential errors.

On the old irrigated and perspective irrigation files due to ignoring the process of global climate warming irrigation regime do not consider growing needs for water. On the contrary, on the Yavan valley files recommended regimes of the irrigation are connected with over expenditure of water resources. For example, last specifications on regimes of the irrigation Yavan valley on annual average means of humidity coefficient (0.35) to the category of droughty areas. However, for last 20 years evaporation in a valley has decreased almost on 300 mm (17 %) and the quantity of precipitation has risen on 70 mm (11 %) and humidity coefficient up to 0.45. Hence present irrigating norms for cultivation of the middle-fibrous cotton in Yavan valley is 1100 m^3/ha and 3000 m^3/ha for Lucerne are overestimated. Calculations show, that unproductive losses of water only on two valleys are made more 60 mln.m^3.

Conclusion

To ensure the dynamic development of agriculture and thus food security must take action adaptation, including an increase in the productivity of agricultural land and irrigation water, and most importantly the involvement of modern achievements of biotechnology - selection of high productivity crops steady to stress and climate change

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